A Discussion on Primitive Pythagorean Triples and Primitive Pythagorean Primes

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Abstract. There are many methods available to obtain Pythagorean triples (PT). The general observation shows that most of them do not produce all the Primitive Pythagorean Triples (PPT) or they produce all triples but with repetition. This paper highlights a structured approach to finding all PPTs and subsequently all triples, such that there is no repetition in the triples. In the process, an interesting relation between the sum of lengths of the legs of a primitive right triangle with prime numbers is established. Later, a Fundamental Theorem of Primitive Pythagorean Primes (PPP) is conjectured, which is in many ways similar to the Fundamental Theorem of Arithmetic, along with interesting propositions on PPPs derived from $r=1$ condition.

1. Introduction

A PT [1] is a set of $a, b, c \in Z^+$ such that $a^2 + b^2 = c^2$. A PT is called a PPT, if a, b, c are relatively prime. The most general formula to compute PTs, the Euclid's method, states that, $\forall\, m, n, k \in Z^+$ and $m > n$, (a, b, c) forms a PT, where $a = k(m^2 - n^2)$, $b = kmn$, $c = k(m^2+n^2)$. But this method duplicates the PTs, as we could see for $k = 1$, $m = 9$, $n = 3$, $k = 9$, $m = 3$, $n = 1$ and $k = 18$, $m = 2$, $n = 1$, the PTs generated are (72, 54, 90), (54, 72, 90) and (54, 72, 90) respectively.

Only a few algorithms exist to find all the PPTs and PTs without repetition. The first to show this was F J M Barning [2], who developed three matrices, such that when any of the three matrices is multiplied on the right by a column matrix such that the elements of the matrix form a PT, then the result is another column matrix whose elements are a different PT. If the initial input is a PPT, then the result is also a PPT. Thus each PPT could generate three PPTs. All PPTs are produced this way from the PPT (3, 4, 5), and no PPT appears more than once. A different tree was found by Price [3] which again uses three matrices to produce triples.

There are a few other methods to produce all PPTs, like the method which uses Euclid's formula for generating all PPTS [4] and Mitchell's formula [5] which makes use of two parameters to generate the tree of triples. The results obtained from these methods become identical to the tree produced from Barning's method. William J Spezeski [6] recently presented a method which generates all triples exactly twice without repetition.

Though the methods used in obtaining ternary trees generate all the PPTs and also without repetition, but the fate of obtaining a PPT, depends on another PPT. i.e. we cannot obtain an individual PPT (referred to as spigot PPT). The formula discussed below is capable of generating all the PPTs individually without repetition based on integer parameters. Also, the classification of PPTs, as per the PPT table, allows us to explore and study wide range of properties adhered to PPTs. There are several analytical and numerical methods [11] for estimating prime numbers. Veracity and eloquence of such concepts and proofs motivated the work documented in this paper.

2. A Structured Approach

An approach in the area of PTs shows that all the primitive triples without repetition can be found by the relation between the even leg and the hypotenuse. Hence a formula has been devised based on the following PPT table and observations.

Throughout the paper, let us assume that $\forall$ PPTs $[a, b, c]$, a = odd leg, b = even leg, c = hypotenuse (Fig. 1).

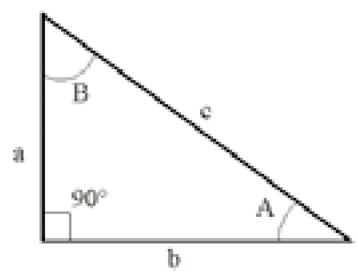

Fig. 1. Right angled triangle.

Theorem 2.1. *The difference in lengths of hypotenuse and the even leg in a PPT should always be equal to (odd number)^2.*

Proof. Let $l = (2k-1)$ be an odd number $\forall k \in Z^+$

$$\Rightarrow c - b = l$$
$$\Rightarrow \sqrt{a^2+b^2} - b = l$$
$$\Rightarrow \sqrt{a^2+b^2} = b + l$$

squaring,

$$a^2 + b^2 = b^2 + 2bl + l^2$$
$$\Rightarrow a^2 = l(2b+l)$$
$$\Rightarrow a = \sqrt{l(2b+l)}$$

since LHS $\in Z^+$, RHS $\in Z^+$

$$\Rightarrow \sqrt{l}\sqrt{2b+l} \in Z^+$$
$$\Rightarrow \sqrt{l} \in Z^+$$

and

$$\sqrt{2b+l} \in Z^+$$
$$\Rightarrow l = j^2$$

where, j is an odd number. In other words, $l = (2k-1)^2$, so that l is the square of an odd number.

Based on Theorem 1, the PPTs were classified and the following PPT table was obtained.

2.1. PPT table

	$r=1$	$r=2$	$r=3$	$r=4$	$r=5$
$s=1$	(3,4,5)	(15,8,17)	(35,12,37)	(63,16,65)	(99,20,101)
$s=2$	(5,12,13)	(21,20,29)	(45,28,53)	(77,36,85)	(117,44,125)
$s=3$	(7,24,25)	(27,36,45)	(55,48,73)	(91,60,109)	(135,72,153)
$s=4$	(9,40,41)	(33,56,65)	(65,72,97)	(105,88,137)	(153,104,185)
$s=5$	(11,60,61)	(39,80,89)	(75,100,125)	(119,120,169)	(171,140,221)
$s=6$	(13,84,85)	(45,108,117)	(85,132,157)	(133,156,205)	(189,180,261)
$s=7$	(15,92,93)	(51,140,149)	(95,168,193)	(147,196,245)	(207,224,305)

2.2. Derivation

In order to obtain a general formula to represent all PPTs, let us first obtain a representation for a, the odd leg.

Let $a_{r,s}$ denote the odd leg corresponding to column r and row s of the PPT table. Consider the first row of the PPT table.

Let $3, 15, 35, 63, 99 \ldots$ be denoted by a series $S_{r,1}$ as:

$$S_{r,1} = 3 + 15 + 35 + 63 + 99 + \ldots a_{r,1}.$$

Now,

$$S_{r,1} = 3 + 15 + 35 + 63 + 99 + \ldots a_{r,1} \quad (1)$$

$$S_{r,1} = \quad 3 + 15 + 35 + 63 + 99 + \ldots a_{r,1}. \quad (2)$$

(1)–(2)

$$\Rightarrow 0 = 3 + 12 + 20 + 28 + 36 + \ldots (a_{r,1} - a_{r-1,1}) - a_{r,1}$$
$$\Rightarrow a_{r,1} = 3 + 4(3 + 5 + 7 + 9 + \ldots (2r-1))$$
$$\Rightarrow a_{r,1} = 3 + 4(1 + 3 + 5 + 7 + 9 + \ldots (2r-1) - 1)$$
$$\Rightarrow a_{r,1} = 3 + 4(r^2 - 1). \quad (3)$$

As we traverse down any column, we can find that $a_{r,s} = a_{r,s-1} + \tau$, where $a_{r,s} \in$ row s, $a_{r,s-1} \in$ row $s-1$ of the prescribed column and τ, the constant difference between them. Clearly,

$$a_{r,s} = a_{r,1} + (s-1)\tau. \quad (4)$$

Also $\tau = 2(2r-1)$, which can be obtained by solving the series 2, 6, 10, 14 ... for the rth term. Therefore from (3) and (4) choose, $a = 4r^2 - 1 + 2(2r-1)(s-1)$.

Theorem 2.2. *$\forall s \in$ multiples of prime factors of $(2r-1)$, the PTs obtained at s have a highest common factor which is the square of the prime factor whose multiple is taken by s. i.e. Let $2r - 1 = {\omega_0}^{\alpha_0}{\omega_1}^{\alpha_1}{\omega_2}^{\alpha_2}\cdots{\omega_k}^{\alpha_k}\cdots{\omega_n}^{\alpha_n}$ be the prime factorization of $2r-1$. If $s = m\omega_k, m \in Z^+$ then at s, the highest common factor of PT $(a, b, c) = {\omega_k}^2$.*

Proof. We know that,

$$a = 4r^2 - 1 + 2(2r-1)(s-1).$$

When $s = m\omega_k$,

$$a = 4r^2 - 1 + 2(2r-1)(m\omega_k - 1)$$
$$= (2r-1)(2r+1) + 2(2r-1)(m\omega_k - 1)$$

$$= (2r-1)[(2r+1)+2m\omega_k-2]$$
$$= (2r-1)[(2r-1)+2m\omega_k]$$
$$= ({\omega_0}^{\alpha_0}{\omega_1}^{\alpha_1}{\omega_2}^{\alpha_2}\cdots{\omega_k}^{\alpha_k}\cdots{\omega_n}^{\alpha_n})$$
$$\times[{\omega_0}^{\alpha_0}{\omega_1}^{\alpha_1}{\omega_2}^{\alpha_2}\cdots{\omega_k}^{\alpha_k}\cdots{\omega_n}^{\alpha_n}+2m\omega_k]$$
$$= {\omega_k}^2({\omega_0}^{\alpha_0}{\omega_1}^{\alpha_1}{\omega_2}^{\alpha_2}\cdots{\omega_k}^{\alpha_k-1}\cdots{\omega_n}^{\alpha_n})$$
$$\times[{\omega_0}^{\alpha_0}{\omega_1}^{\alpha_1}{\omega_2}^{\alpha_2}\cdots{\omega_k}^{\alpha_k-1}\cdots{\omega_n}^{\alpha_n}+2m]$$
$$\Rightarrow a = {\omega_k}^2\beta$$

where, $\beta = ({\omega_0}^{\alpha_0}{\omega_1}^{\alpha_1}{\omega_2}^{\alpha_2}\cdots{\omega_k}^{\alpha_k-1}\cdots{\omega_n}^{\alpha_n})\times[{\omega_0}^{\alpha_0}{\omega_1}^{\alpha_1}{\omega_2}^{\alpha_2}\cdots{\omega_k}^{\alpha_k-1}\cdots{\omega_n}^{\alpha_n}+2m]$. Similarly, b and c also will have a common factor ${\omega_k}^2$.

Note: When $s = m{\omega_k}^2 = q\omega_{k+\theta}^2$, $\theta \in Z^+$, then clearly, $\omega_{k+\theta}^2$ is the highest factor of PT (a, b, c).

2.3. Observations

1. We could spot the appearance of non-primitive PTs at some positions of the PPT table. These positions correspond to the value of s becoming equal to multiples of prime factors of $2r-1$. By restricting s at these values, we can obtain a PPT formula and beyond reasonable doubt a PT formula, which produces all PPTs and PTs without repetition.

Based on the above theorems, derivations and observations the formula is obtained.

3. Formula

$$\forall,\quad \phi = 4r^2-1+2(2r-1)[s-1]$$

where,
$r \in Z^+$
$s \in Z^+-$ {multiples of prime factors of $2r-1$}.
The PPTs $[a, b, c]$ are:

$$\left[\phi, \frac{\phi^2-(2r-1)^4}{2(2r-1)^2}, \frac{\phi^2+(2r-1)^4}{2(2r-1)^2}\right].$$

All the possible PTs can be found out without repetition by adding a multiplier to the above formula like this: The PTs $[a', b', c'] = m[a, b, c]$ are:

$$m\left[\phi, \frac{\phi^2-(2r-1)^4}{2(2r-1)^2}, \frac{\phi^2+(2r-1)^4}{2(2r-1)^2}\right]$$

where, $m \in Z^+$.

Proof. By definition,

$$a^2+b^2=c^2$$

from Theorem 1, we know that $c = b+(2r-1)^2$

$$\Rightarrow a^2+b^2 = b^2+2b(2r-1)^2+(2r-1)^4$$
$$\Rightarrow a^2 = 2b(2r-1)^2+(2r-1)^4$$
$$\Rightarrow b = \frac{a^2-(2r-1)^4}{2(2r-1)^2}$$
$$c = b+(2r-1)^2 = \frac{a^2+(2r-1)^4}{2(2r-1)^2}.$$

Hence, replacing a by ϕ, the complete PPT is found.

3.1. Other form

The other way of expressing the above formula is, the PPTs $[a, b, c]$ are:

$$[4r^2-1+2(2r-1)(s-1), 2s(2r+s-1), 2s(2r+s-1)+(2r-1)^2]$$

where,
$r \in Z^+$
$s \in Z^+-$ {multiples of primefactors of $(2r-1)$}.
The Pythagorean Triples $[a'b', c'] = m[a, b, c]$ are:

$$m[4r^2-1+2(2r-1)(s-1), 2s(2r+s-1),$$
$$2s(2r+s-1)+(2r-1)^2]$$

where, $m \in Z^+$.

3.2. Verification

To verify the correctness of the formula, the following examples are considered.

1. Example 1: $r=1$, $s=1, m=1 \Longleftrightarrow (3,4,5)$
2. Example 2: $r=1$, $s=1$, $m=7$ (*multiplier*) $\Longleftrightarrow$ $(21,28,35)$
3. Example 3: $r=3$, $s=4, m=1 \Longleftrightarrow (65,72,97)$
4. Example 4: $r=5$, $s=3$. Here, value of s is invalid, s cannot be equal to 3. s must not be equal to multiples of prime factors of $(2r-1)$. Here $(2r-1) = (2(5)-1) = 9$. Prime factors of 9 is 3. Hence s in this case must not be equal to 3 or multiples of 3.
5. Example 5: $r=12$, $s=23$, $m=4$ (multiplier). Here, value of s is invalid, s cannot be equal to 23. S must not be equal to multiples of prime factors of $(2r-1)$. Here $(2r-1) = (2(12)-1) = 23$. Prime factors of 23 (prime factor of a prime number is the number itself) is 23. Hence s in this case must not be equal to 23 or multiples of 23.
6. Example 6: $r=17$, $s=29$, $m=3$ (multiplier) $\Longleftrightarrow (3004,3596,4685)$.

The new set of formulae produce all the PPTs and of course all the PTs, without repetition. PPTs can be used in cryptography as random sequences and for the generation of keys. Also by including all values for s, i.e. by making $s \in Z^+$, we can obtain a dynamically changing pattern/code for extremely high security, apart from that, the triples are now better classified/organized for study as can see from the following properties.

4. Two Prime Entries

In any PPT one of the three entries must be even, while the other two entries namely, hypotenuse and the odd leg must be odd. Since the later entries are odd, there is a chance that both the entries could be prime numbers. Also it is conjectured that there are infinitely many such instances wherein both, the odd leg and the hypotenuse are prime numbers [7]. By Schinzel and Sierpinski's Hypothesis H [8] we then expect to see infinitely many triples with two prime entries.

Theorem 4.1. *All the PPTs, wherein both, the odd leg and the hypotenuse are prime numbers must fall in the first column of the PPT table. i.e. If $r = 1$, then $\forall s \in Z^+$, we can find infinitely many two prime entries. And it is impossible to find a two prime entry $\forall r \neq 1$.*

Proof. The PPTs $\forall r = 1$, $s \in Z^+ -$ {multiples of prime factors of $2r - 1$} are

$$[a, b, c] = (2s + 1, 2s(s + 1), 2s(s + 1) + 1).$$

The odd leg $2s + 1$ may or may not be a prime number.

The PPTs $\forall r \in Z^+$, $s \in Z^+ -$ {multiples of prime factors of $2r - 1$} are:

$$\left[\phi, \frac{\phi^2 - (2r-1)^4}{2(2r-1)^2}, \frac{\phi^2 + (2r-1)^4}{2(2r-1)^2}\right]$$

where,

$$\phi = 4r^2 - 1 + 2(2r - 1)[s - 1].$$

Here,

$$\frac{\phi}{2r-1} = 2(r + s) - 1 \in Z^+.$$

Hence, $\forall r \neq 1$, there cannot be a two prime entry.

Theorem 4.2. *All the PPTs, where $a^2 + b^2 = c^2$ and $b + c = a^2$, must lie in the first column of PPT table.*

Proof. From the equations, $a^2 + b^2 = c^2$, $b + c = a^2$, we have $(b + c)[1 + (b - c)] = 0$. Since sides cannot be negative, $b \neq -c$, so, $c - b = 1$. Hence, proved.

5. Primitive Hypotenuse Proposition

Proposition 5.1. *Let $t = 2r - 1$, $r \in Z^+$, then t is prime, iff*

$$\sum_{s=1}^{\infty}[t(2s + t) + 2s^2] = \sum_{k=1}^{t-1}(ik)^2$$

where, $s, k \in Z^+$; $i = \sqrt{-1}$.

Proof. From 3.1, we know that any hypotenuse of a PPT could be represented by the formula $c = 2s(2r + s - 1) + (2r - 1)^2$, $r, s \in Z^+$, $s \neq$ multiples of prime factors of $2r - 1$. Clearly,

$$c = 2s(2r + s - 1) + (2r - 1)^2$$

$$\Rightarrow c = s^2 + s^2 + 2s(2r - 1) + (2r - 1)^2$$

$$\Rightarrow c = s^2 + [s + (2r - 1)]^2.$$

Now consider $t = 2r - 1$ and $c_r = \sum_{s=1}^{\infty} c$, $s \neq$ multiples of prime factors of $2r - 1$ where, c_r denotes the sum of all the Primitive hypotenuses belonging to column r of the PPT table. For instance, $c_1 = 1^2 + (2)^2 + 2^2 + (3)^2 + 3^2 + (4)^2 + \cdots$ and $c_2 = 1^2 + (4)^2 + 2^2 + (5)^2 + 4^2 + (7)^2 + \cdots$. Before proceeding further, it is to be noted that if t is prime, then $t = \omega_0$ (from Theorem 2) and if t is composite, then $\exists\, \omega_0$, such that $\omega_0 < t$. When we have t as a prime number, then

$$c = s^2 + [s + t]^2$$

and

$$c_r = 1^2 + 2^2 + 3^2 + \ldots \omega_0^2 + (\omega_0 + 1)^2 + \cdots + (1 + t)^2 + (2 + t)^2 + (3 + t)2 + \cdots$$

$$\Rightarrow c_r = 1^2 + 2^2 + 3^2 + \ldots t^2 + (t + 1)^2 + \cdots + (1 + t)^2 + (2 + t)^2 + (3 + t)2 + \cdots$$

$$\Rightarrow c_r = 1^2 + 2^2 + 3^2 + \cdots + 1^2 + 2^2 + 3^2 + \cdots - (1^2 + 2^2 + 3^2 + \ldots t^2) - (t^2 + (2t)^2) - ((2t)^2 + (3t)^2) + \cdots$$

$$\Rightarrow c_r = 1^2 + 2^2 + 3^2 + \cdots + 1^2 + 2^2 + 3^2 + \cdots - (1^2 + 2^2 + 3^2 + \ldots t^2) - (t^2 + 2t^2[-1 + 1^2 + 2^2 + 3^2 + \cdots])$$

$$\Rightarrow c_r = 0 + \cdots + 0 + \cdots - (1^2 + 2^2 + 3^2 + \ldots t^2) - (t^2 + 2t^2[-1 + 0]).$$

Since, $\zeta(-2) = 0$, where ζ denotes the Riemann zeta function [9]. Therefore upon simplification,

$$c_r = -(1^2 + 2^2 + 3^2 + \ldots (t-1)^2)$$

$$\Rightarrow c_r = \sum_{k=1}^{t-1} (ik)^2.$$

Hence, proved.

6. Fundamental Theorem of PPP

The fundamental theorem of arithmetic [1] states that for every natural number $n > 1$, n is either a prime number or is composite such that it could be expressed as a product of prime numbers raised to suitable powers. For example, 3 is prime, whereas 6 is composite such that $6 = 2 \times 3$. Consider the complete set of odd numbers $2s + 1$, $s \in Z^+$. This could be written as $2(s+1)^1 - 1$. It has to be noted that, any number of this sequence is either prime or is composite such that its prime factors again belong to the same sequence. For instance, if we consider $s = 3$, we have $2(3 + 1) - 1 = 7$, which is prime and it belongs to the sequence. Now consider $s = 4$, we have $2(4 + 1) - 1 = 9$, which is composite. The prime factors of 9 is 3, which again belongs to the same sequence.

Let $P = a + b$, where a and b are the odd leg and even leg of a PPT respectively. If P is prime, then it is termed as a Primitive Pythagorean Prime (PPP). The fundamental theorem of arithmetic obviously holds for PPPs. The interesting thing is that if P is not a prime number, then it is expressible as the product of prime numbers raised to suitable powers which are all again PPPs.

6.1. Conjecture 1

It is conjectured that, every P is either a prime number, or is a composite number which could be expressed as a product of PPPs. i.e. consider the complete set of $P = a + b$, which could be represented as, $2[r+(2s-1)]^2-(2s-1)^2$, $r \neq$ multiples of prime factors of $2s - 1$. It is conjectured that every number of this sequence is either prime or could be expressed as the product of prime factors raised to suitable powers such that the prime factors in turn belong to the same sequence. The same property is satisfied by the set of odd numbers and the set of natural numbers and also by the followed sequences:

$\delta_1 = |a \pm b| - \{1\}$
$\delta_2 = |a \pm 2b| - \{1\}$
$\delta_4 = |a \pm 4b| - \{1\}$
$\delta_5 = c$

where, a, b, c are the odd leg, the even leg, and the hypotenuse of a PPT respectively.

It is not known if more of such sequences exist or not which produce numbers having unique prime factorization and also that the prime factors belonging to the same sequence, ignoring the elementary sequence generated by p^k, where p is a prime number raised to an integer k.

Proposition 6.1. *Consider all PPTs whose hypotenuse and even leg differ by a unit length. i.e. consider all PPTs, corresponding to $r = 1$, $s \in Z^+$ (column one of PPT table). For any value of s, there cannot be more than three consecutive PPPs, i.e. the maximum number of PPPs associated with s in sequence is three OR if s yields a PPP, and if $s + 1$ yields a PPP, and if $s + 2$ yields a PPP, then $s + 3$ cannot yield a PPP. Also,*

$$s + 2 \equiv 0 \ (mod\ 7)$$

i.e. $s + 2\ (mod\ 7) = 0$, or $s + 2$ is an integral multiple of 7.

Note: The only exception for divisibility by 7, is the set, $s = 1$, $s + 1 = 2$, $s + 2 = 3$.

Proof. When we consider the first column of the PPT table, the PPPs belonging to this column will be $P = 2(s+1)^2 - 1$. Let us say that P is not a prime and has 7 as a prime factor. In that case let,

$$P/7 = x$$

then clearly,

$$s = \left(\frac{7x+1}{2}\right)^{\frac{1}{2}} - 1.$$

Solving RHS to obtain $s \in Z^+$, one can observe that we obtain $s = 1, 4, 8, 11, 15, 18, 22, 25, 29\ldots$ so $\forall s$ taking the values of the preceding sequence, we can affirmatively say, that P will not be prime and also that P has a prime factor 7. As we can see, this sequence could be split and represented as $1 + 7s$ and $4 + 7s$. Therefore, we could conclude that the maximum number of PPPs that could occur together can never be more than 3. And also, wherever the three PPPs occur together, the value of s corresponding to the third PPT is a multiple of 7.

Proposition 6.2. *Consider the first column of the PPT table. The sum of the values of s corresponding to which P encounters the first two occurrences of a prime factor G must be equal to $G - 2$.*

Proof. Given, $2(s_1+1)^2-1 = P_1$ and $2(s_2+1)^2-1 = P_2$. Let

$$P_1 \equiv 0 \pmod{G}.$$

Need to show that at $s = G - 2 - s1$,

$$P_2 \equiv 0 \pmod{G}$$

$$P_2 = 2(s_2 + 1)^2 - 1$$

$$\Rightarrow P_2 = 2(G - 2 - s_1 + 1)^2 - 1$$

$$= 2(G - (s_1 + 1))^2$$

$$= 2G^2 - 4G(s_1 + 1) + 2(s_1 + 1)^2 - 1$$

$$= 2G^2 - 4G(s_1 + 1) + P_1.$$

Therefore, *G divides* P_2. Hence proved. Also it is evident that at $s = s_1 + hG$ and $s = s_2 + hG$, $h \in Z^+$

$$P \equiv 0 \pmod{G}.$$

7. Conclusion

We have seen a structured approach for obtaining all the PPTs without repetition using a PPT table. This approach has made it easy for us to explore various properties of PPTs. We were able to grasp the conjecture on the PPPs which could be of much use in the future considering their close resemblance with the properties of positive integers subject to Fundamental Theorem of Arithmetic.

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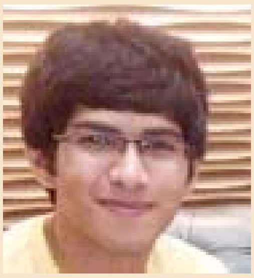

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