Interview with Hidetoshi Fukagawa

Sujatha Ramdorai

Hidetoshi Fukagawa

Sujatha Ramdorai: Tell us a little about yourself.

Hidetoshi Fukagawa: I was born in Kitakyushu city, Fukuoka prefecture in 1943, and graduated in 1967 from the Mathematics Department in Yamaguchi University. I served as a high school teacher of mathematics in Aichi Prefecture during 1967–2004, having taught at six high schools. I married my wife Miyako in 1972 and obtained my PhD in Mathematics Education from the Bulgaria Academy of Sciences in 1996. I retired as a high school teacher in 2004, and since then have been teaching courses on "Mathematics and Mathematics Education" at several universities, part of the time. Currently, I give lectures at Daidou University and Kogakkan University. In my free time, I enjoy growing vegetables in the fields. I have published several books and articles related to mathematics, especially Sangaku, and they are listed below chronologically.

[1989] With Dan Pedoe, *Japanese Temple Geometry Problems: Sangaku* (Winnipeg University, CBRC, Canada).

[1991] *Japan Geometric — How Many Problems Can You Solve*, in Japanese (Morikita Shuppan Publication).

[1994] *Japanese Mathematics* (two volumes), in Japanese (Morikita Shuppan Publication).

[1998] *Sangaku and Japanese Mathematics*, in Japanese (Morikita Shuppan Publication).

[2002] With John F Rigby, *Traditional Japanese Mathematics Problems of the 18th and 19th Centuries* (SCT-publishing, Singapore).

[2002] Translation. Robert Geretshlager, *Geometric Origami*, translated by Fukagawa Hidetoshi into Japanese (Morikita Shuppan Publication).

[2005] Supervised in "Big Sangaku Exhibition" held at the Nagoya City Science Museum, sponsored by Asahi Shinbun Company.

[2008] With Tony Rothman, *Sacred Mathematics* (Princeton University Press).

[2011] With Kitaoka Yoshiyuki and Kawamura Tsukasa, *Fundamental Calculus for Engineering*, university text, in Japanese (Gakujyutsu Syuppan).

[2013] *Elementary Mathematics*, in Japanese (Kogakkan University Press).

SR: How did you first hear of the Sangaku?

HF: Being a math teacher, I looked for materials by examining the history of mathematics, so that students could have interest in mathematics. In Japan, at that time, mathematics during the Edo period was specific to that period and had no worth for high school, and so no one had been studying it. Its content is very low and unavailable for high-school mathematics, in general. Only Greek mathematics, or Abel or Galois story were investigated by all math teachers without looking at the mathematical content. Mathematical contents of

western mathematics history are too professional for high school students, and they were difficult.

When I served as a teacher at first in Handa high school in Aichi Prefecture, I met a linguist teacher Kunihiko Sakakibara who was a prominent linguist. One day, he brought a dirty old book of Japanese mathematics to me, saying, "It's Edo period math book and I don't know the content, can you decode it?" It was the first Edo period math book for me. I had never seen authentic Edo period math books until then. I said to him. "It is generally believed that mathematics during Edo period had totally useless, low level content. So don't expect too much", and began the translation work. I had studied Chinese, just as studying Latin is for anyone interested in history of the Western world. I finished the translation of the book six months later and I was very much surprised.

Although they didn't know western mathematics, mathematicians during the Edo period used "Partial derivative" or "Power series". I was startled and so started to study traditional Japanese mathematics. There were many Japanese mathematics books. In my study, I found a word, "Wooden tablet on which Japanese mathematics problems recorded" in some book. What is a wooden plank with mathematical problems? Then I found a board kept unnoticed to any modern mathematicians. There was a wooden plank in existence in Aichi prefecture on which mathematics problems were written during the Edo period. I was surprised and investigated, and found that local historians had reported about it. They found the content difficult for them, because they were not mathematicians, and the plank was recorded as a mere cultural property. Some Sangaku survived in Mie and Aichi prefectures, as mere cultural properties, are preserved. This was the start of my love for Sangaku.

SR: Have you tried using the Sangaku problems and method of solving them in the classroom?

HF: Most Sangaku problems were introduced only as a cultural asset and were difficult for high school students. The high school attached to Aichi educational College was my third school. One day, I was going to Nagasaki for the school excursion with my students. Before departure, I had translated Sangaku problems of Nagasaki Suwa Shrine into modern language and handed the problems to my class students. On the day before they set out for travel, somehow during my lunch break I went to my classroom and found five or six students debating about Suwa shrine Sangaku problems. From this time, I was convinced that easy Sangaku problems are best for students. But Japanese high school students are busy in everyday classes, and they have no extra time to study Sangaku. I once was a lecturer at the Education Centre for Aichi Prefecture's high school teachers, and introduced the importance of Sangaku, from 1980 to 1992. Now, I am talking on Sangaku in my lecture at Nagoya University and some other Universities in mathematics education course.

SR: You can obviously mix this with history, culture, and politics. Just as in the introductory portions of your book. Have you tried this form of mixing the problems with story-telling in the class? What are the reactions of the students?

HF: Traditional Japanese mathematics is based on Chinese mathematics and is independent from Western mathematics. In 17th century, Japan was exposed to the danger of the invasion from the West. Finding out that other Asian Nations were invaded by the West, the Tokugawa Government prohibited other countries that had traded with Japan from intervening in domestic matters. The Tokugawa government was sensitive to and restricted the invasion of particularly religious or scientific information. In the 17th and 18th centuries, in Japan, science and culture took roots on their own. This is Edo culture and we give too much appreciation to it, that is, traditional Japanese culture. The culture includes mathematics too. The culture developed, because the Tokugawa Government avoided war, aimed at agricultural growth, to be a rich country. People lived in not so poor but rich environment, and so the culture developed. In high school, I have been speaking, "Peace developed culture and peace is so important". Sangaku is a product of popular culture. I always emphasised this in the classroom. There were a lot of people who enjoyed mathematics during the Edo period.

SR: Are there other allusions to Sangaku in Japanese texts or folklore?

HF: Studies of mathematics during the Edo period were done as cultural property only and not in the view of mathematics. In a Japanese history textbook there was only one line about it. The Sangaku hadn't touched everyone. Historians couldn't understand the high mathematical level of the Sangaku. Sangaku was reported as material left from history and worthless for mathematics education. I focused on studying Sangaku, analysed the contents of mathematical research, but

it was a heresy, and has been ignored by historians. However, I noted its contents, and continued to study their style. Mr Haruki Abe, who was a reporter of Asahi news company, became interested in my research. He introduced Sangaku that I studied in the Asahi newspaper. Researchers studied the materials that were left, panicked and began examining the Sangaku since then. The historians then wrote about Sangaku in the history textbooks of high school in Japan.

Sangaku didn't appear in any Japan high school history textbooks until 40 years ago, but is now described in Japan history textbooks. However, the mathematical content of Sangaku is very difficult to introduce.

SR: Is there any evidence that the Japanese were aware of Mathematical knowledge from other civilisations around the world at that time?

HF: Western science, translated into Chinese, was imported to Japan during the Edo period. Calendar was important for the Tokugawa government and the knowledge of the Western calendar was coming into Japan. Especially the calculator Abacus or "Soroban" (just as the modern PC was imported from China), and they used it on many occasions of everyday life. Abacus training rooms were opened in many regions of Japan in the Edo period.

During the 17th–19th centuries in Japan, many small schools teaching reading and writing were opened. Village classrooms sprouted up throughout the country to teach reading, writing and arithmetic, or three "R"s. The classroom was called "Jyuku" or "Terakoya", and played a particularly important role for increasing literacy rate and pushing up the general level of education of the common people.

Schools were run even privately, and the size varied widely from 10 to 1,000 students. Poor samurais, priests and any intelligentsia could become a teacher of the school easily, and could get some money. These schools were private, not public. Therefore, many people were strong in calculation, reading and writing. At some Jyuku, advanced mathematics was studied, and people who loved math studied there. In such schools, mathematics was just really fun in ordinary life. In some provinces where there were mathematics lovers, the wooden tablets, on which geometry problems or the land surveying results were written, were hung in the shrine, and thanked God. Thus many elementary schools proliferated in 18th and 19th centuries of Japan. Number of cram schools in the Edo period was more than 80,000. (Refer to http://library.u-gakugei.ac.jp.)

The literacy rate in Japan at that time was so high and said to be more than 70%. During the 18th and 19th centuries, it is said that the literacy rate in London was about 20%, and in Paris, 10%. However, in Edo period, it was 70–86%. It was easy to run such schools, even in the home garden. Samurais who lost their jobs and some intellectuals easily ran such schools. The Government thought that the influence of literacy was also useful in letting people know laws, so did not put restrictions on such schools. Mathematicians of the Edo era were good at computation of geometric problems. Greek mathematics depended on logic, but Japanese mathematics used complex calculations. Logical thinking was not born in Japan. But computational geometric problems thrived and left behind great material.

SR: Tell us a little about your experience in collaborating with Rothman for the book.

HF: Rothman and I have never met each other until now. However, we published the book. Mathematicians who joined Sangaku tour and asked me to be the guide were surprised that we as co-authors have not seen each other. I started to research Sangaku when I was a high school teacher. But nobody recognised my work. During the Edo period, mathematicians studied hard geometric problems and most of Sangaku problems were on geometry. They used computational geometry without logic. Geometric logic was not used in Japan. Until 40 years ago, Sangaku mathematics had been ignored, that is, Sangaku was a forgotten world. All mathematicians in Japan at that time thought the mathematics during the Edo period had no value. But plenty of problems, from the view of a high school math teacher, are suitable for high school students.

I wanted to introduce this forgotten mathematics to the world and so acted. I guess it was 1984; my first job was buying a book on how to write in English. I have never written an English letter until then. My favourite language was only Japanese (this is a joke). If I hadn't come across Sangaku, then I wouldn't have needed to study English. I studied how to write English letters for the first time at the age of 42. I sent letters, in which Sangaku geometric theorems were written, to ten foreign mathematicians who had interest in history and geometry all over the world. But nobody sent me any reply. These were letters I wrote to foreign countries for the first time. Sangaku inspired me towards such

behaviour because I was mad about Sangaku. Two months later, my wife shouted, an air mail arrived from abroad. I received a letter from abroad for the first time. The letter was from Dan Pedoe (1910–1998) of the University of Minnesota. He was impressed by my letter, "Sangaku is a wonderful world, so we two will research it". In 1988, I know that Dan Pedoe struggled in publishing the Sangaku. When he introduced the theorem on an ellipse of Sangaku in his lecture in Australia, one participant sent another solution to me. Dan Pedoe consulted about publishing "Sangaku" with Australian Mathematical Society. The answer to this was Ralph Stanton of the University of Winnipeg in Canada. *Japanese Temple Geometry: Sangaku* was published by CBRC, Winnipeg in 1989, thanks to him. This book contains interesting geometry problems. It is my first book, dedicated to Australian Mathematical Society.

The book was read by many people. Thanks to this book, Sangaku was introduced to the world for the first time. Tony Rothman especially had many answers and asked questions about this book. The interaction with Rothman is the longest one. He solved many problems. And I helped him write his article in *Scientific American* in 1998. This article was organised by Tony about 5 years ago. From then on, Tony and I continued to discuss on Sangaku problems over a long period. There happened more interesting things. Tony moved to Princeton University, deepened the acquaintance with the eminent physicist Freeman Dyson. Freeman Dyson was a student, to whom Dan Pedoe had taught geometry in London. So Dyson cooperated with me after Dan Pedoe requested him to help me. In 1994, I bought a fax machine. The first fax mail was not from any Japanese but from Princeton University, Freeman Dyson's recommendation to my new book in Japan. Collaboration between Freeman Dyson, Tony Rothman and myself enabled *Sacred Mathematics* to be issued from Princeton University in 2008. Tony Rothman drew all of about 300 figures, rewrote my manuscript and changed the sequential order of the contents. He did most of the work. My English is still not good and I am learning English from my English teacher Mr Takeshi Taniguchi, my friend. Even now, my English is still not so good.

SR: How have the Japanese received your book? I think there was a recent Japanese movie on Yamaguchi Kanzan and the Sangaku. Was it related to your work in any way?

HF: After my book *Japanese Temple Geometry Problems: Sangaku* with Dan Pedoe was published in Canada, I thought it would be easy for me to publish the Japanese version since I originally wrote the manuscript in Japanese. However, it was not easy to find a publisher in Japan. I wasn't so famous then. However, Mr Setsuo Tanaka, staff of *Morikita Shuppan*, publisher in Tokyo, was intrigued by my manuscript. In 1991, I managed to publish the Japanese version of my book, *Japanese Temple Geometry Problems* with Morikita publisher. It is said in Japan, that a mathematics book is a success if 3,000 copies are sold. The book I translated sold more than 12,000 copies. One reader who was in hospital sent me a letter, in which he wrote "I have a lot of time to solve many interesting problems in the book. Many thanks for your book". I was the first person to introduce Sangaku to the world as mathematics. In 2008, Japanese version of Fukagawa Hidetoshi and Tony Rothman's *Sacred Mathematics* by Princeton University was published by the same publisher *Morikita Shuppan*. After this book, historians got interested in the study of Sangaku. A movie "Tenchi Mei Satsu" featuring the strategist of Edo period was made in 2012. I was not involved in this film. I have not seen this movie. In this movie, Yamaguchi Kanzan has nothing to do with story of this film. Yamaguchi's diary has not yet been published in modern Japanese language.

SR: How has the world outside reacted to your extraordinary findings [Research]?

HF: The late Professor Shiko Iwata, prominent geometer in Japan, knew that the mathematical contents of Sangaku were very good. Iwata, his friend Isao Naoi and I had been studying and analysing traditional Japanese Mathematics very enthusiastically. I thought about studying English, to introduce Sangaku abroad. I sent a letter to ten geometricians of the world, in which beautiful Sangaku theorems were written. Only Dan Pedoe gave me a reply. He was a friend of Leo Sauv (1921–1987), a world renowned geometrician who edited at that time Canada's geometric problem solving journal *Crux Mathematicorum*. Dan Pedoe advised me to contribute the Sangaku problems to *Crux Mathematicorum* of Canada. Leo Sauv edited the Sangaku problem and put the problem 995 in the December 1984 issue. This is the first time that Sangaku was introduced to the larger world. I thereafter provided many problems with this magazine, *Crux Mathematicorum*. I have received letters from people in many countries who read the magazine. Of course, I introduced Sangaku problems to mathematics magazines in Japan prior to

this, but nobody had been interested in the Sangaku world. The book published by Winnipeg University with the help of Dan Pedoe in 1989, *Japanese Temple Geometry* was decisive. Pedoe wrote most of the texts since I was not good at English. He was a prominent geometer and wrote Sangaku problems as "problems of Euclid". Pedoe has spread Sangaku to all over the world. A modern Greek complained to me in his letter, "There is no Geometry other than in Greece". After some years, he came to love Sangaku.

SR: What is the oldest Sangaku that you have come across?

HF: There are 900 Sangaku tablets remaining and the oldest Sangaku was hung in 1683 at Hoshinomiya shrine of Tochigi Prefecture. The surface of this plate was burnt, and cannot be decrypted. But before it was burnt, local junior students had already made a replica, we can see its content. Because we were unable to carry the earliest Sangaku, we carried the replicas when the national Sangaku exhibition was held in Nagoya. I supervised and exhibited it in 2005 at the Nagoya City Science Museum. The replica of the earliest Sangaku is a board 180 cm wide and 90 cm in height. There is actually in the Edo era (1603–1867), 54 years ago before the earliest surviving Sangaku, a recording of an even older Sangaku hung in the shrine of Fukushima Prefecture after 1657. After the Sangaku exhibition in Nagoya, five more tablets were found in Toyama Prefecture. When I held the Sangaku national exhibition, a local historian became interested in the Sangaku and found one new Sangaku. However, in the catastrophic earthquake disaster in the Northeast Japan in 2011, we lost two or three tablets in Fukushima prefecture. Recently in modern children's events, they make small Sangakus freely, in which modern school mathematics, not traditional ones, is written. But they have nothing to do with traditional Japanese Mathematics.

SR: You are clearly passionate about this. What future plans do you have in this regard?

HF: Two weeks ago, I solved problems of a Sangaku tablet dedicated in 1841 at Tashiro shrine in Yoro-Cho of Gifu prefecture, which I visited and gave the analysis to the priest, since nobody had been able to analyse the content of the Sangaku by then. Five problems written on the tablet were so interesting. Three of them were submitted by twelve, thirteen, and eleven year old boys respectively. Two other problems were so difficult to solve that it took me one month to solve the problems. I wanted to show the five problems to undergraduate mathematics students. Sangaku problems often interest people. For example, a geometric problem among birds and flowers, depicted on the ceiling grid in the temple in Nagano prefecture was so nice and I submitted it to the *Crux Mathematicorum*. Now the temple no longer exists.

A British geometrician visited Japan several years ago, and he wanted me to see his solution to the problem and came to my home in Gifu, since he was so excited to find it after some struggles. He found the best answer to this problem. We just met in my home, and I have never been acquainted with him before that. In fact his solution was so good. His name is John F Rigby and his solution is introduced in [2]. Thanks to his invitation, I was given a chance to demonstrate the Sangaku to the teachers of high school who graduated from the University of Wales in 2004.

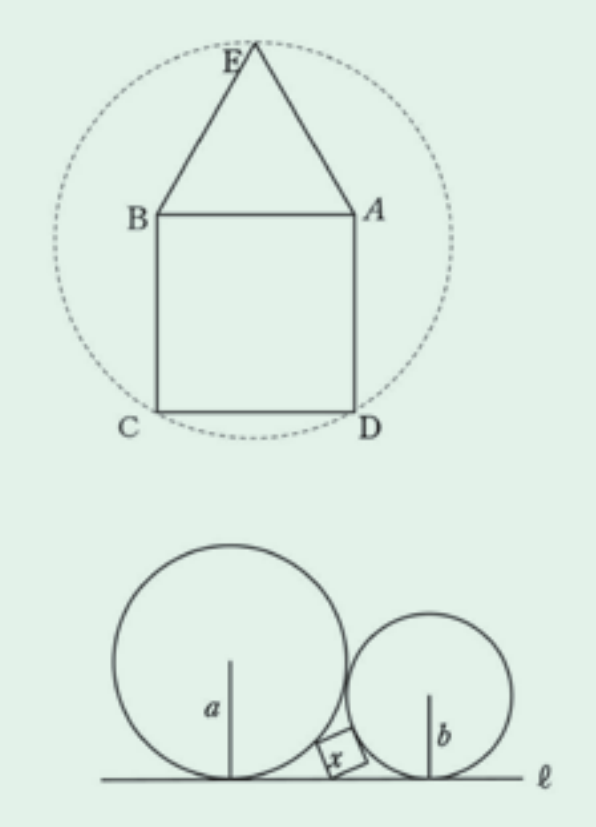
In 2012, the idea of giving some Sangaku tours to the people from other countries was proposed to me. In March 2012, a Swiss mathematician, Professor Emmanuelle Gracchi, and in April, Carsten Cramon of Denmark with 20 people visited me. In July, American mathematician David Clark of Randolph-Macon college, in August, a student Ian Johnston of Boston University, and in September, Rosalie Hosking from New Zealand visited Ogaki Sangaku. Currently I am preparing for a visit of 20 Singapore high school students in August 2013. They are going to visit Sangakus at Atsuta shrine in Nagoya city and Ogaki. Since languages are different, they won't know its contents. One visitor, Professor Peter Wong of Bates College in the US, with more than 20 years of acquaintance, knows the content of Sangaku. I was surprised when I met him for the first time, by his understanding of the content of the Sangaku. However, he couldn't understand or speak Japanese. Generally people don't know the contents of Sangaku and so I would like to translate it, and introduce it to the world. Sangaku is a great material for mathematics education. In September this year, I will visit two universities — Valladorid and Sevilla of Spain, and will give a talk on Sangaku.

SR: Your own personal favourite problem and solution among the Sangakus.

HF: Circles and triangles and rectangles are geometric tools in traditional Japanese mathematics. Let me introduce an interesting simple problem. An equilateral triangle ABE has sides of length 10 units and, an outside

square ABCD has side length 10 units. The problem is to find the radius of the circle that passes through the three points E, C, and D. This is not a Sangaku problem but quoted from the book published in 1877. The other problem has not been solved yet and is recorded in Yamaguchi's diary, which is a Sangaku problem. It was described as "problem not solved yet" in his diary. I think it's pretty hard. I have not been able to solve it. Two externally touching circles of radii a and b lie on the straight line L. Describe a square inscribed in the space bounded by two circles and the line. The side of square x varies. Find the minimum and maximum of x in terms of a and b. I think it's pretty hard.

In 1994, in some conference of Bulgaria, an aged mathematician who sat in the front row at the Conference in Bulgaria asked me a question. Are there unresolved problems in the Sangaku world? Since I had not studied the diary of Yamaguchi at that time, I couldn't answer him. His name was Paul Erdos (1913–1996).



SR: Has this taught you something about traditional schooling or learning in Japan?

HF: Sangaku problems are difficult for high school students and I didn't teach them at my high school. I introduced Sangaku problems in "Mathematics magazines for high school students" in Japan. Other work of mine on Sangaku was to introduce Sangaku to high school math teachers of Aichi prefecture in the workshop during 1980–1992. Currently I am teaching university students of the department of mathematics at Nagoya University, from 1990 till now. Moreover, I have introduced the Sangaku in mathematics education at several universities.

From 1987, in Bulgaria, I provided Sangaku problems on the cover every month for teenage magazines in Bulgarian Education of Mathematics and Informatics until 1992. In the 1987 issue, a student of Bulgaria high school came up with a wonderful solution to a Sangaku problem. This project was run by geometrician Jordan Tavob of Bulgaria Academy of Science.

In 2005, I organised "The National Sangaku exhibition" at Nagoya City Science Museum and exhibited one hundred Sangakus, carrying them to Nagoya by truck. The leading newspaper Asahi Shimbun spent a huge amount of money. Mr Abe Haruki from Asahi Shimbun proposed this exhibition. This large scale exhibition touched many people.

SR: When you compare the solutions of some of the geometric problems and solutions, is there anything strikingly different in the way of approaching a problem in this culture as opposed to say, the Greeks and the geometry developed by the Greeks? Or later in other European countries?

HF: "Wasan", traditional Japanese Mathematics, developed based on the Chinese mathematics. Sangaku world is a part of "Wasan". Computation is the subject there. Contrary to this, Greek mathematical or Western geometry has evolved from logic as the subject. For example, when a triangle was given, Feuer Bach circle has the relation passing through three midpoints of the sides of triangle and nine points of the triangle in western geometry. Wasan mathematicians calculated the radius of the circle touching three circles in terms of three sides. The aims of western mathematics and Japanese mathematics are the relations and calculations respectively. Complex calculation is the main aim of Wasan and Sangaku.

See [1, p. 111]. In [3, Chapter 6, p. 232], you can find factoring a fourth degree equation into two second degree equations. Few Sangaku theorems on the ellipses are found in Western mathematics. See [1, pp. 50–68].

Sangaku problems introduced in [3, pp. 212–216] were calculated redundantly. Immediately after publication in the United States, a good short solution appeared in the article in the USA. After long calculations on circles or spheres contacting problems, Japanese

mathematicians then got a clean result. But it can be gained easily by the modern inversion technique. In 1937, in the west "six ball chain theorems" was published by Nobel prize winner Fredric Soddy, but the same theorem had already been used in Sangaku about 100 years before. Also we find the same analysis diagrams, "Descartes circle theorem" in complete works of Descartes, but it is seen in the old book already used. See [3, p. 286].

SR: Thank you for this wonderful interview, it has been a pleasure interacting with you.

References

[1] H. Fukagawa and D. Pedoe, *Japanese Temple Geometry Problems: Sangaku* (CBRC, Winnipeg University, 1989).

[2] H. Fukagawa and John F. Rigby, *Traditional Japanese Mathematics Problems of the 18th and 19th Centuries* (SCT-publishing, Singapore, 2002).

[3] H. Fukagawa and T. Rothman, *Sacred Mathematics* (Princeton University Press, 2008).

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