The SASTRA Ramanujan Prize — Its Origins and Its Winners

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The SASTRA Ramanujan Prize is a $10,000 annual award given to mathematicians not exceeding the age of 32 for path-breaking contributions in areas influenced by the Indian mathematical genius Srinivasa Ramanujan. The prize has been unusually effective in recognizing extremely gifted mathematicians at an early stage in their careers, who have gone on to accomplish even greater things in mathematics and be awarded prizes with hallowed traditions such as the Fields Medal. This is due to the enthusiastic support from leading mathematicians around the world and the calibre of the winners. The age limit of 32 is because Ramanujan lived only for 32 years, and in that brief life span made revolutionary contributions; so the challenge for the prize candidates is to show what they have achieved in that same time frame! The way the prize was conceived and launched is an incredible story which I will relate here. I will then briefly describe some major aspects of the works of the winners, and a few of their biographical highlights.

The Origins

The district of Tanjore (=*Tanjavur*) in the state of Tamil Nadu in South India, has been a seat of culture for several centuries. Tanjore has produced some of the greatest composers and performers of South Indian classical music. Tanjore is well known for art in various forms and has the greatest concentration of Hindu temples. Ramanujan was born in Erode on Dec 22, 1887, but it was in the Tanjore region steeped in culture that he lived in the town of Kumbakonam until he completed high school.

During the second half of the 20th century, Ramanujan's humble home in Kumbakonam from where a thousand theorems emerged, was in a dilapidated condition. Even though in India many events and programs were held regularly in Ramanujan's memory, including the grand Ramanujan Centenary celebrations in December 1987, nothing was done for the renovation of this historic home. One of the most significant developments in the worldwide effort to preserve and honor the legacy of Ramanujan is the purchase in 2003 of Ramanujan's home in Kumbakonam by SASTRA University to maintain it as a museum. This purchase had far reaching consequences because it led to the involvement of a university in the preservation of Ramanujan's legacy for posterity.

The Shanmugha Arts, Science, Technology and Research Academy (SASTRA), is a private university in the town of Tanjore after which the district is named. SASTRA was founded in 1984 and has grown by leaps and bounds. Admission is very competitive, and SASTRA has succeeded in attracting some of the brightest students in India.

SASTRA has renovated Ramanujan's home beautifully without altering its structure or design. The only modification was to add a bust of Ramanujan in the living room. In connection with the purchase of Ramanujan's home, SASTRA University opened a branch campus in Kumbakonam in 2003, called the Srinivasa Ramanujan Center. SASTRA has a museum there in which several important letters, photographs and documents related to Ramanujan are displayed. A visit to Kumbakonam just to see Ramanujan's home and this museum will be worthwhile and inspiring.

One afternoon in September 2003, I received a phone call from S Swaminathan, then a graduate student at the University of Virginia (now a Dean at SASTRA). He introduced himself as the son of Vice-Chancellor R Sethuraman of SASTRA University, gave me some background about SASTRA and said that to mark the occasion of the purchase of Ramanujan's home, SASTRA would be conducting an international conference that year in December at the Kumbakonam campus. The President of India, Dr Abdul Kalam, had agreed

*This is the original and slightly expanded version of an article under the same title that appeared in *The Notices of the American Mathematical Society* **66** (2019) 64–72. Permission granted by the AMS is gratefully acknowledged.

Krishna Alladi being introduced to the President of India Dr Abdul Kalam during the First Ramanujan Conference at SASTRA University on Dec 20, 2003. To the left of Krishna are George Andrews (Penn State) and Noam Elkies (Harvard).

to inaugurate this conference on Dec 20, 2003 and declare Ramanujan's home as a national treasure. Swaminathan invited me to bring a team of mathematicians from abroad to the conference, and said that SASTRA would cover all their expenses including international travel. I felt that what I heard over the phone was incredible because international airfare is rarely paid for participants of mathematical conferences in India. But on checking I realized that something important was taking place. So I called George Andrews of The Pennsylvania State University, the world's foremost authority in the theory of partitions and on Ramanujan's work, told him about SASTRA, and invited him to come to the conference. He said that based on my assurance, he would come. In fact Andrews gave the opening lecture of the conference as well as the concluding Ramanujan Commemmoration Lecture on December 22, Ramanujan's birthday. At the valedictory function, the participants suggested that SASTRA should conduct conferences annually around Ramanujan's birthday on different areas of mathematics influenced by Ramanujan. Dean Vaidhyasubramaniam of SASTRA agreed to this suggestion and invited me to help organize these annual conferences. I have done this annually since 2003, and so have also had the pleasure of being in Ramanujan's hometown each year in December thus making it an annual pilgrimage for me (see article 25 in [1], pp. 153–160).

For the second SASTRA Ramanujan Conference in 2004, the Vice-Chancellor invited me to inaugurate it and also deliver the concluding Ramanujan Commemmoration Lecture. I was honored to receive such an invitation and I accepted it happily.

Conference inaugurations in India are usually elaborate and grand ceremonies starting with the traditional "lighting of the lamp" by the Chief Guest, with the lamp representing enlightenment through knowledge. There was a large gathering in the auditorium, and as we walked up to the stage to be seated before the official start of the ceremony, the Vice-Chancellor whispered to me and said "I wish to give $10,000 annually for a worthy cause in the name of Ramanujan. You please decide how it should be used and announce it in your inaugural speech." I was pleasantly shocked by this and said "This is very generous of you, but are you asking me to announce this in the next few minutes without consulting anybody?" He said "Yes, consulting people, or going through a Committee, will cause unnecessary delays, and so I want you to come to decision right now and announce it." Fortunately, my inaugural speech was preceded by the lamp lighting ceremony and the customary elaborate welcome of the audience and the introduction of the Chief Guest (me) as is common in India, and so I had at least 15 minutes to think about it as all this was going on! When I got up to speak, I mentioned this very generous offer from the Vice-Chancellor, and suggested that a SASTRA Ramanujan Prize be created and that $10,000 be given to a mathematician not exceeding the age of 32 for outstanding contributions to areas influenced by Ramanujan. I pointed out that the

Fields Medal is given only to those under 40 years of age, and the SASTRA Prize would be given to even younger mathematicians, some of whom may later win the Fields Medal or other well established major prizes. The Vice-Chancellor liked this suggestion very much, and at the conclusion of the inaugural ceremony, in thanking me, he confirmed that the prize would be launched the next year. He then turned towards me and said "I request you to be the Chair of the Prize Committee." So that is how the SASTRA Ramanujan Prize was conceived and launched, and how I got involved with the Prize. All this happened within the span of an hour on Dec 20, 2004 during the inauguration.

Upon return to the United States, I called George Andrews and told him this incredible story of the launch of the prize and said that for this to be a success, I would need the support of the mathematical community. In particular, I wanted his presence on the Committee for the first year. He was suprised and happy to hear this and agreed to serve on the Prize Committee. Over the years, eminent mathematicians have supported the prize by either serving on the Prize Committee, making nominations, or writing letters evaluating the work of the nominees. The prize is given annually at SASTRA University, Kumbakonam, during an international conference held around Ramanujan's birthday, except in the year 2012 when it was given in New Delhi (see the 2012 Prize below).

The Prize Committee consists of six mathematicians besides me. They all vote, and I serve as the non-voting chair. I vote only to break a tie. Three of the six step down each year, and three new members join. Thus each voting member on the Committee serves a two year term.

The Winners

The 2005 Prizes: Manjul Bhargava of Princeton University and Kannan Soundararajan of the University of Michigan came out on top as the strongest young mathematicians in the areas of algebraic number theory and analytic number theory respectively, and were equally ranked 1 by the Committee.

One of the pioneering discoveries of Gauss was the composition law for binary quadratic forms. Introducing several new and unexpected ideas, Bhargava broke an impasse since the time of Gauss, and established in his 2001 PhD thesis written under the direction of Andrew Wiles of Princeton University, composition laws for higher degree forms. Bhargava applied these to solve new cases of one of the fundamental questions of number theory, that of the asymptotic enumeration of number fields of a given degree. This had far reaching consequences on various problems in algebraic number theory. He published the results in his thesis in a series of papers in the *Annals of Mathematics*. Bhargava's lecture at SASTRA University upon accepting the prize was on a different topic; he announced his joint work with Jonathan Hanke on the solution of the problem of determining all universal quadratic forms — a problem whose origin can be traced back to Ramanujan.

Kannan Soundararajan had made spectacular contributions to many parts of analytic number theory, most notably pertaining to the Riemann zeta function and Dirichlet L-functions — especially on the distribution and location of their zeros. He also established deep results in random matrix theory, which has fundamental connections with prime number theory. With Hugh Montgomery of the University of Michigan, he showed that prime numbers in short intervals are distributed normally, but with a variance that differs from classical heuristics. In his PhD thesis written under the direction of Peter Sarnak at Princeton University, he showed that 7/8ths of the quadratic L-functions have no zeros at the critical point $s = 1/2$ which provided strong evidence for a certain conjecture of Chowla. Along with Brian Conrey, he had shown that a positive proportion of Dirichlet L-functions have no zeros on the real axis within the critical strip.

The Prize Committee felt that both candidates deserved the full award. The SASTRA Vice-Chancellor generously agreed to the recommendation of the Committee that Bhargava and Soundararajan be awarded two full prizes, and that it should not be split. Thus the SASTRA Ramanujan Prize could not have had a better start (see Ken Ono's article [3] on the award of the first SASTRA Ramanujan Prizes, and mine ([1], pp. 161–166) as well.)

Two years prior to winning the SASTRA Prize, Bhargava was appointed Full Professor at Princeton at the age of 29, the youngest at that rank.

Kannan Soundararajan (then at Michigan) receiving the First SASTRA Ramanujan Prize on Dec 20, 2005 from Dr Arabinda Mitra, Director, Department of Science and Technology, India. (Second from Left) Manjul Bhargava (Princeton) who had received the same prize a few minutes earlier, looks on. Next to Dr Mitra is SASTRA Vice-Chancellor R Sethuraman, and next to Soundararajan is Krishna Alladi.

Bhargava had graduated from Harvard in 1996 with the highest honors and received the Hoopes Prize. That year he was awarded the Morgan Prize of the AMS for undergraduate research. For his revolutionary PhD work, he was awarded the Blumenthal Prize of the AMS in January 2005. Just prior to coming to SASTRA, he received the Clay Prize at Oxford University. Bhargava continued producing fundamental work after receiving the SASTRA Ramanujan prize, especially relating to the average rank of elliptic curves. He was recognized with the AMS Cole Prize in 2008 and the Fields Medal in 2014.

Soundararajan hails from Madras (now Chennai), India, where he began his research in number theory while in high school. He won a Silver Medal in the International Mathematics Olympiad in 1991. As an undergraduate at the University of Michigan, he received the First Morgan Prize of the AMS in 1995. After obtaining his PhD from Princeton, for outstanding contributions to analytic number theory he was awarded the Salem Prize in 2003. When Soundararajan received the SASTRA Prize, he was a tenured faculty member at Michigan, and in the next year, was appointed Full Professor at Stanford. He has continued to produce work of outstanding quality and subsequently, he was recognized with the Infosys Prize and the Ostrowski Prize, both in 2011.

The 2006 Prize: Thirty one-year old Terence Tao of UCLA had made far reaching contributions to diverse areas of mathematics such as number theory, harmonic analysis, partial differential equations and ergodic theory. He is widely regarded as one of the most influential mathematicians of our times.

One of Tao's most notable contributions was for the Kakeya Problem in higher dimensions. One aspect of the problem is to determine the fractal dimension of a set obtained by rotating a needle in n-dimensional space. In a joint work with Nets Katz, Isabella Laba and others, he improved all previously known estimates for the fractal dimension with ingenious combinatorial ideas. Another of Tao's seminal contributions was his joint work with Ben Green on long arithmetic progressions of prime numbers. One of the deepest results in this area is due to the Hungarian mathematician Szemeredi who showed that any set of positive density will have arbitrarily long arithmetic progressions. Subsequently, Tim Gowers of Cambridge University gave a very different proof of Szemeredi's theorem. This result of Szemeredi does not apply to the set of primes which is of zero density. By combining the ideas of Gowers along with tools from ergodic theory, Green and Tao proved the sensational result that there are arbitrarily long arithmetic progressions of primes.

Krishna Alladi reads the citation before the 2006 SASTRA Ramanujan Prize is presented to Terence Tao (standing, middle) of UCLA.

Yet another fundamental contribution of Tao concerns the sum-product problem due to Paul Erdös and Szemeredi. Roughly speaking, this problem states that either the sum set or the product set of a set of N numbers must be large. Tao was the first to recognize the significance of this problem in combinatorial number theory and harmonic analysis. In collaboration with Nets Katz and Jean Bourgain, Tao made important generalizations which led to breakthroughs in harmonic analysis and number theory. Tao has also made inroads into physics through his study of wave maps that occur in Einstein's theory of relativity, and his work on Schrödinger equations.

Tao was born in Adelaide, Australia in 1975 and lived there until 1992. He did his BSc (Honors) and MSc at Flinders University in South Australia. He went to Princeton in 1992 for his PhD which he completed in 1996 under the direction of Elias Stein. Honors have been awarded to him in a steady stream. Prior to winning the Fields Medal and the SASTRA Ramanujan Prize in 2006, he was awarded the Salem Prize (2000), the Bocher Prize of the AMS (2002), the AMS Conant Prize (2005), and the Fields Medal in August 2006. More recently, he received the 2014 Breakthrough Prize. He currently holds the James and Carol Collins Chair at UCLA.

The 2007 Prize: Ben Green of Cambridge University had made phenomenal contributions to several fundamental problems in number theory by himself and in collaboration with Terence Tao. Green's PhD thesis at Cambridge is a collection of several outstanding papers. In one of these papers he solved the Cameron–Erdös conjecture which is a bound on the number of sum-free sets of

2007 SASTRA Prize Winner Ben Green (then at Cambridge University) seated on the windowsill of the bedroom at Ramanujan's home. The cot there was the only one for Ramanujan's family. As a boy, Ramanujan used to sit on the windowsill and do his "sums" watching the passers-by on the street.

positive integers up to a given number N. Prior to collaborating with Tao, Green had also established an important result that any set of primes with relative positive density would contain infinitely many arithmetic progessions of length 3. It was this paper of Green's in the 2005 *Annals of Mathematics* that caught the attention of Tao and which led to their collaborative and definitive result on arbitrarily long arithmetic progressions among the primes. Subsequently Green and Tao also collaborated in extending the Hardy–Ramanujan–Littlewood Circle Method by bringing in methods from ergodic theory.

Green was born in Bristol, England in 1977. He went to Cambridge University to do his BA (1995–98) and continued to do his PhD there (1999–2002) under the direction of Fields Medalist Tim Gowers. He received the 2001 Smith Prize for his PhD work. In 2005, Green was appointed Hershel Smith Professor at Cambridge University, and was awarded the Salem Prize and the Whitehead Prize that year. He was elected Fellow of the Royal Society in 2010 and was awarded the

Akshay Venkatesh (Stanford), 2008 SASTRA Prize Winner, garlanding the statue of Ramanujan at SASTRA University.

Sylvester Medal in 2014. He is currently Waynflete Professor of Pure Mathematics at Oxford University.

The 2008 Prize: Akshay Venkatesh of Stanford University was creating waves in the mathematical world, by making powerful contributions to diverse areas, by himself and with a host of collaborators. His 2006 paper with H Helfgott contained striking and original ideas, and provided the first non-trivial upper bound for the 3-torsion in class groups of quadratic fields. His joint work with Jordan Ellenberg on representing integral quadratic forms by quadratic forms had its roots in the work of Ramanujan.

An important and difficult problem in number theory is to asymptotically count number fields according to their discriminant. This is a generalization of the classical problem of determining the relation between the number of rational or integral solutions of a polynomial equation in several variables and the coordinates of the solutions. The case up to degree 5 had been solved by Manjul Bhargava. For large degrees, Ellenberg and Venkatesh provided the first major improvement over bounds in the earlier work of Wolfgang Schmidt and thus broke an impasse of 30 years. Also of great importance was Venkatesh's work on sub-convexity of automorphic L-functions, and his joint work with E Lindenstrauss which settled a famous conjecture of Peter Sarnak's concerning locally symmetric spaces. One of Venkatesh's most impressive achievements was his individual work on subconvexity of automorphic L-functions. The problem of subconvex bounds at the center of the critical strip for L-functions is very important. Venkatesh provided a novel and more direct way of establishing subconvexity in numerous cases that went beyond the foundational work of Hardy–Littlewood–Weyl and of current leading researchers as well.

2009 SASTRA Prize Winner Kathrin Bringmann (University of Cologne) lighting the ceremonial lamp to mark the opening of the Ramanujan Conference.

Venkatesh who is of South Indian descent, was born in New Delhi, India in 1981 but was raised in Perth, Australia. He showed his brilliance early by winning medals in both the International Physics and Mathematics Olympiads at the age of 12. His entry into research began in 1998 when he joined Princeton for his PhD under the direction of Peter Sarnak. On completing his PhD in 2002, he became Moore Instructor at MIT and then a Clay Research Fellow. He received the Salem Prize in 2007 and was appointed Full Professor at Stanford the next year. After the SASTRA Ramanujan Prize, he has been awarded the Infosys Prize in 2016 and the Ostrowski Prize in 2017. In 2018 he was awarded the Fields Medal and appointed to the permanent faculty at the Institute for Advanced Study, Princeton, New Jersey.

The 2009 Prize: The 2009 Prize was awarded to Kathrin Bringmann for work related to Ramanujan's mock theta functions. Ramanujan's discovery of the mock theta functions and the identities he communicated about them in his last letter to G H Hardy in 1920 just a few months before his death, are among his deepest contributions. Mock theta functions are objects which are like the classical theta functions in their shape, but are not modular forms, yet their coefficients can be calculated with a degree of precision comparable to what can be done for functions that can be expressed in terms of theta functions. No one knew the exact relationship between theta functions and mock theta functions, and indeed determining this relationship was one of the tantalizing puzzles of mathematics. Following a lead provided by Sander Zwegers, Kathrin Bringmann and her post-doctoral mentor Ken Ono provided the key to unlock this mystery by showing how mock theta functions are intimately connected with harmonic Maass forms.

Bringmann was led to the investigation of mock theta functions when she did her post-doctoral work under Ken Ono at the University of Wisconsin. Bringmann and Ono, inspired by the work of Zagier and Zwegers, wrote a fundamental paper that appeared in the *Annals of Mathematics*, in which they showed that Ramanujan's 22 mock theta functions are special cases of infinite families of Maass forms of weight 1/2. This and other works by Bringmann, Ono, and their collaborators, has resolved the puzzle surrounding mock theta functions and led to an understanding of their fundamental role in the mathematical landscape.

Kathrin Bringmann was born in 1977 in Muenster, Germany. She obtained a Diploma in Mathematics with top honors from the University of Wuerzburg, Germany, in 2003. She then went to the University of Heidelberg where she received her PhD under the direction of Winfried Kohnen in 2004. Following that, she became Van Vleck Assistant Professor in Wisconsin, where her collaboration with Ken Ono began. After a brief period in Minnesota, she joined the University of Cologne, Germany, as Professor. In 2009, prior to receiving the SASTRA Prize, she was awarded the prestigious Krupp Prize, which provides her support at the University of Cologne to conduct a comprehensive program in number theory with several PhD students, post-docs and visitors.

The 2010 Prize: Wei Zhang, who did his doctoral work under Professor Shouwu Zhang at Columbia University, had made seminal contributions by himself and in collaboration with others to a broad range of areas in mathematics including number theory, automorphic forms, trace formulas, *L*-functions, representation theory and algebraic geometry.

In 1997 Steve Kudla constructed a family of cycles on Shimura varieties, and conjectured that their generating functions are actually Siegel modular forms. The proof of the Kudla conjecture for cycles of codimension-1 is a major theorem of Fields Medalist, Richard Borcherds. In his PhD

2010 SASTRA Prize Winner Wei Zhang (then at Harvard) seated (on the right) along with his former PhD advisor Shouwu Zhang (Columbia) at the entrance to Ramanujan's home in Kumbakonam.

thesis, Zhang established conditionally, among other things, a generalization of Borcherds' result for higher dimensions and in that process essentially settled the Kudla conjecture. Zhang's thesis opened up major lines of research and led to a significant collaboration with Xinyi Yuan and Shouwu Zhang. The three established an arithmetic analogue of a theorem of Waldspurger that connects integral periods to values of *L*-functions. In addition, Wei Zhang by himself had done outstanding work on relative trace formulas, and on Shimura varieties.

Wei Zhang hails from the People's Republic of China where he was born in 1981. After obtaining his Bachelor's degree from Peking University in 2004, he joined Columbia University to do his PhD. When he heard of the Kudla Conjecture as a first year graduate student, he started working on it, and in one year, had mastered the techniques needed to attack it. After his PhD, he became Post-Doctoral Fellow at Harvard. When Zhang received the SASTRA Prize, he had become

Benjamin Pierce Instructor at Harvard. In 2017 he was awarded the New Horizons Prize in Mathematics. He is currently a Full Professor at Columbia University.

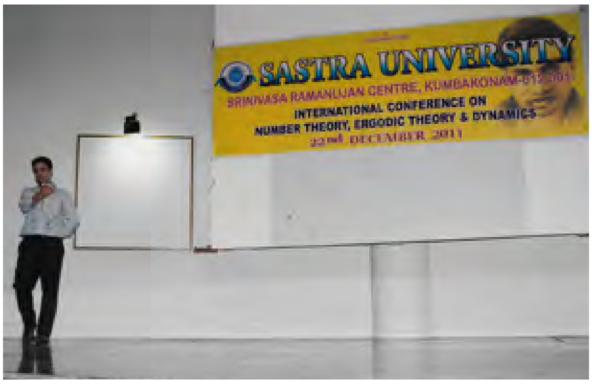
The 2011 Prize: Roman Holowinsky of Ohio State University had made significant contributions at the interface of analytic number theory and the theory of modular forms. Along with Kannan Soundararajan (winner of the SASTRA Ramanujan Prize in 2005), he solved an important case of the famous Quantum Unique Ergodicity (QUE) Conjecture. This was a spectacular achievement.

In 1991, Zeev Rudnick and Peter Sarnak formulated the QUE Conjecture which in its general form concerns the correspondence principle for quantizations of chaotic systems. One aspect of the problem is to understand how waves are influenced by the geometry of their enclosure. Rudnick and Sarnak conjectured that for sufficiently chaotic systems, if the surface has negative curvature, then the high frequency quantum wave functions are uniformly distributed within the domain. The modular domain in number theory is one of the most important examples, and for this case, Holowinsky and Soundararajan solved the holomorphic QUE Conjecture.

The manner in which this solution came about is amazing. By a study of Hecke eigenvalues and an ingenious application of the sieve, Holowinsky obtained critical estimates for shifted convolution sums and this almost settled the holomorphic QUE Conjecture for the modular domain except in certain cases where the corresponding L-functions behave abnormally. Simultaneously, Soundararajan who approached the problem from an entirely different direction, was able to confirm the conjecture in several cases, and noticed that the exceptional cases not fitting Holowinsky's approach were covered by his techniques. Thus by combining the approaches of Holowinsky and Soundararajan, the holomorphic QUE Conjecture was fully resolved in the modular case.

Holowinsky also wrote two fundamental papers — on a sieve method for shifted convolution sums, and on sieving for mass equidistribution.

Holowinsky was born on July 26, 1979. He obtained his Bachelors degree from Rutgers University in 2001, where he continued to do his PhD under the direction of Henryk Iwaniec. Already in his PhD thesis he made major advances



2011 SASTRA Prize Winner Roman Holowinsky (Ohio State) delivering the Ramanujan Commemmoration Lecture on Dec 22, Ramanujan's birthday.

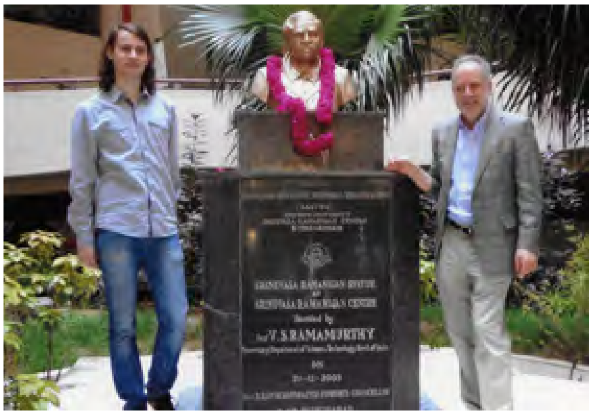
towards the QUE Conjecture. He held postdoctoral positions at the Institute for Advanced Study, the Fields Institute and the University of Toronto before joining the permanent faculty at Ohio State University. In 2011, in addition to the SASTRA Ramanujan Prize, he was awarded a Sloan Fellowship.

The 2012 Prize: Zhiwei Yun of Stanford University had made fundamental contributions to areas that lie at the interface of representation theory, algebraic geometry and number theory. Yun's PhD thesis on global Springer theory at Princeton University written under the direction of Robert MacPherson of The Institute for Advanced Study, opened up new vistas in the Langlands program. Springer theory is the study of Weyl group actions on the cohomology of certain subvarieties of the flag manifold called Springer fibers. Yun's global Springer theory deals with Hitchin fibers instead of Springer fibers which he used to determine the actions of affine Weyl groups on cohomology.

Bao-Châu Ngô was awarded the 2010 Fields Medal for his proof of the Fundamental Lemma in the Langlands Program. Yun made a breakthrough in the study of the Fundamental Lemma formulated by Jacquet and Rallis in their program of proving the Gross–Prasad conjecture on relative trace formulas. Yun's understanding of Hitchin fibrations enabled him to reduce the Jacquet–Rallis Fundamental Lemma to a cohomological property of the Hitchin fibration. In addition, Yun's work on the uniform construction of motives with exceptional Galois groups is considered to be very fundamental.

Since 2012 was the 125th Birth Anniversary of Ramanujan, Yun was awarded the SASTRA Prize in India's capital, New Delhi, at a conference

Zhiwei Yun (MIT and Stanford) receiving the 2012 SASTRA Ramanujan Prize from Minister of State Jitin Prasada at the University of Delhi on December 22. SASTRA Vice-Chancellor Sethuraman and Krishna Alladi are looking on. [Photo courtesy: SASTRA University.]



(L to R) 2013 SASTRA Prize Winner Peter Scholze (University of Bonn) and his former PhD Advisor Michael Rappoport (Bonn) standing next to the statue of Ramanujan at SASTRA University.

organized by the National Board for Higher Mathematics (of India), and co-sponsored by SASTRA University and Delhi University. That was the only year when this prize was not awarded in Kumbakonam.

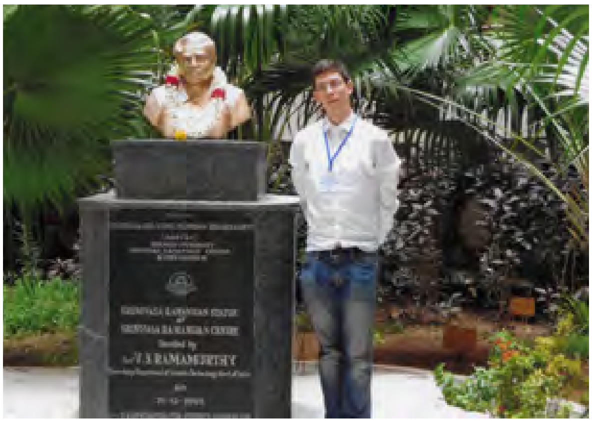
Zhiwei Yun was born in Changzhou, China, in 1982. He showed his flair for mathematics early by winning the Gold Medal in the International Mathematics Olympiad held in Korea in 2000. He obtained his bachelor's degree from Peking University in 2000 and joined Princeton University where he received his PhD in 2009. When Yun received the SASTRA Prize, he had just been appointed to the permanent faculty at Stanford University after having completed his term as a Moore Instructor at MIT. In 2018 he was a recipient of the New Horizons in Mathematics Prize.

The 2013 Prize: Peter Scholze of the University of Bonn had made revolutionary contributions to arithmetic algebraic geometry and the theory of automorphic forms. Already in his Masters thesis at Bonn, Scholze gave a new proof of the Local Langlands Conjecture for general linear groups based on a novel approach to calculate the zeta function of certain algebraic varieties. While this was groundbreaking, his PhD thesis at Bonn under the direction of Michael Rappoport was a much bigger breakthrough: he developed a new p-adic machine called *perfectoid spaces* and used it brilliantly to prove a significant part of the weighted monodromy conjecture due to Deligne, thereby breaking an impasse of more than 30 years. Scholze extended his theory of perfectoid spaces to develop a Hodge theory for rigid analytic spaces over p-adic ground fields, generalizing a theory due to Faltings for algebraic varieties.

Scholze was born in Dresden, Germany, in 1987. He was a winner of three gold medals and one silver medal at the International Mathematics Olympiads. He was made Full Professor in Bonn soon after his PhD. Scholze was the youngest winner of the SASTRA Prize at the age of 25. Following that, he has been recognized with the AMS Cole Prize (2015), the Fermat Prize (2015), the Ostrowski Prize (2015), the Liebniz Prize (2016), and the Fields Medal in 2018.

The 2014 Prize: James Maynard of Oxford University, England, and of the University of Montreal, had made outstanding contributions to some of the most famous problems on prime numbers. He obtained the strongest results at the time on the celebrated prime twins conjecture by showing that the gap between consecutive primes does not exceed 600 infinitely often. Not only did he significantly improve the path-breaking work of Goldston, Pintz, Yildirim, and Zhang, but he achieved it with ingenious methods which were simpler than those used by others.

A generalization of the prime twins conjecture is the prime k-tuples conjecture which states that an admissible collection of k linear functions will simultaneously take k prime values infinitely often. In the last 100 years, several partial results towards the k-tuples conjecture were obtained by replacing prime values with "almost primes", namely numbers with a bounded number of prime factors. Another major achievement of Maynard in his doctoral thesis and in his postdoctoral work in Montreal was to significantly



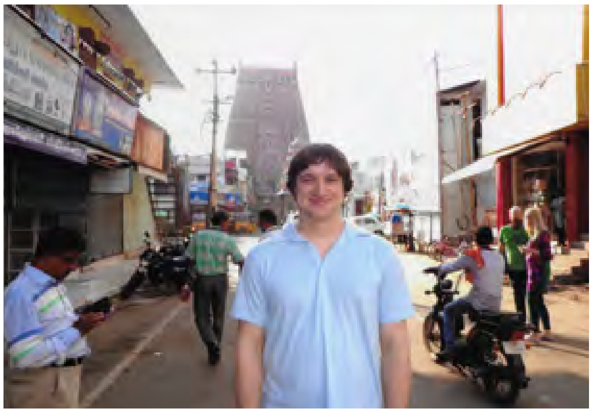
James Maynard (Oxford University), the 2014 SASTRA Prize Winner, standing next to the Ramanujan statue at SASTRA University.

improve on the work of earlier researchers on *k*-tuples of almost primes.

Just before receiving the SASTRA Prize, Maynard announced the solution to the famous $10,000 problem of Erdös on large gaps between primes. This was simultaneously announced by Kevin Ford, Ben Green, Sergei Konyagin, and Terence Tao, but Maynard's method was different and simpler. Maynard's results and methods have led to a resurgence of activity worldwide in prime number theory.

Maynard was born in Chelmsford, England, on June 10, 1987. He obtained his BA and Masters from Cambridge Unibersity in 2009, and his PhD from Oxford in 2013 under the direction of D R Heath-Brown. Following that, he became a post-doctoral fellow at the University of Montreal in 2013–14 under Andrew Granville. After the SASTRA Prize, he received the Whitehead Prize in 2015 and the EMS Prize in 2016. He is currently a professor at Magdalen College, Oxford University.

The 2015 Prize: Jacob Tsimerman of the University of Toronto had made deep and original contributions to diverse parts of number theory, most notably to the André-Oort Conjecture, which states that special subsets of Shimura varieties that are obtained as Zariski closures of special points, are finite unions of Shimura varieties. Shimura varieties are special algebraic varieties (such as moduli spaces of abelian varieties) that arise as quotients of suitable complex domains by arithmetic groups. Yves André initially stated this conjecture for one-dimenional subvarieties, and subsequently Frans Oort proposed that it should hold more generally. A major achievement of Tsimerman in his Princeton PhD thesis of 2011 written under the direction of Peter Sarnak



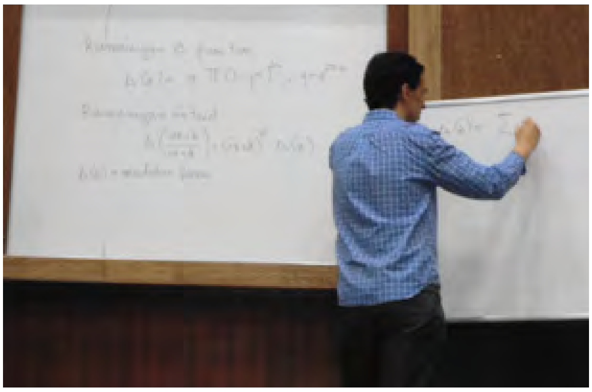
2015 SASTRA Prize Winner Jacob Tsimerman (University of Toronto) standing outside Ramanujan's home. The majestic Gopuram (= entrance tower) of the Sarangapani Temple, where Ramanujan and family regularly worshipped, is seen in the background.

was to obtain certain unconditional bounds up to dimension 6. Another important result in his thesis was to answer in the affirmative a question of Nick Katz and Oort's whether there exists an abelian variety over the set of all algebraic numbers which is not isogenous to the Jacobian of a stable algebraic curve over the algebraic numbers.

Just before receiving the SASTRA Prize, Tsimerman gave a proof of the André-Oort Conjecture for the moduli spaces of principally polarized abelian varieties of any dimension — a result that was sought after for a long time.

Tsimerman was born in Kazan, Russia, on April 26, 1988. His family first moved to Israel, and then to Canada in 1996 where he particpated in mathematical competitions from the age of 9. In 2003 and 2004 he represented Canada in the International Mathematics Olympiads and won gold medals both times. After completing his PhD at Princeton in 2011, he was conferred an AMS Centennial Fellowship. He held a post-doctoral position at Harvard and in 2014 joined the permanent faculty at the University of Toronto where he is presently.

The 2016 Prize: This was shared by Kaisa Matomaki of the University of Turku, Finland, and Maksym Radziwill of McGill University, Canada, and Rutgers University. Their revolutionary collaborative work on multiplicative functions in short intervals shocked the mathematical community by going well beyond what could be proved previously even assuming the Riemann Hypothesis, and opened the door for a series of breakthroughs on some notoriously difficult questions such as the Erdös discrepancy problem



Maksym Radziwill (McGill University) giving the Ramanujan Commemmoration Lecture on his joint work with Kaisa Matomaki with whom he shared the 2016 SASTRA Prize.

and Chowla's conjecture, previously believed to be well beyond reach. Their stunning work on multiplicative functions in short intervals is best illustrated in terms of the Liouville lambda function which takes value 1 when an integer has an even number of prime factors (counted with multiplicity), and value −1 when an integer has an odd number of prime factors. The statement that the lambda function takes values 1 and −1 with asymptotically equal frequency is equivalent to the Prime Number Theorem; more refined statements on the relative error in this equal frequency are related to the Riemann Hypothesis. Such equal distribution results were known also for short intervals, namely intervals $[x, x + h]$, where h is a fractional power (<1) of x. The Riemann Hypothesis implies that powers of h larger than 1/2 will work. Instead of every interval of length h, if we require only "almost all" intervals of length h, it was known that h could be made as small as the 1/6th power of x, and as small as $\log x$ by assuming the Riemann Hypothesis. Matomaki and Radziwill shocked the world, by showing unconditionally that equal frequency holds almost always as long as h tends to infinity with x. Their methods are expected to transform the subject in a major way.

Sarvadaman Chowla conjectured that if any k collection of values of 1 and −1 are given in any order, then the lambda function will take that sequence of values at k consecutive integers with asymptotic frequency $1/2^k$. This conjecture is yet unsolved. In 2016, Matomaki, Radziwill, and Terence Tao proved that when $k = 3$, each of the eight sign choices occur with positive proportion.

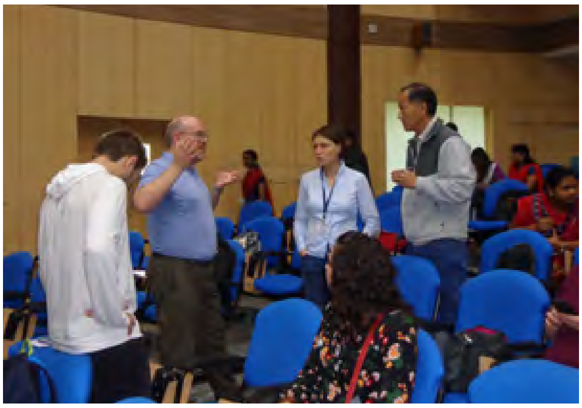
Kaisa Matomaki was born in Nakkila, Finland, on April 30, 1985. While in high school, she won the first prize in the national mathematical competition for Finnish students. She did her Masters at the University of Turku and received the Ernst Lindelöf Award for the best Masters thesis in Finland in 2005. After completing her PhD at the Royal Holloway College, London in 2009 under the direction of Glynn Harman, she returned to the University of Turku where she is presently.

Kaisa Matomaki [Photo Courtesy: Pekka Matomaki.]

Maksym Radziwill was born in Moscow, Russia on February 24, 1988. In 1991 his family first moved to Poland, and then in 2006, to Canada. After writing a superb undergraduate thesis at McGill University in 2009 under the guidance of Andrew Granville, he joined Stanford University where he completed his PhD in 2013 under the supervision of Kannan Soundararajan. He was at the Institute for Advanced Study in 2013–14, and was Hill Assistant Professor at Rutgers during 2014–17, before joining the permanent faculty at McGill University, where he is presently.

The 2017 Prize: Maryna Viazovska of the Swiss Federal Institute of Technology, Lausanne, was awarded the 2017 SASTRA Prize for her stunning solution in dimension 8 of the sphere packing problem, and for her equally impressive joint work with Henry Cohn, Abhinav Kumar, Stephen D Miller, and Danylo Radchenko, resolving the sphere packing problem in dimension 24, by building upon her fundamental ideas in dimension 8.

The sphere packing problem has a long and illustrious history. Johannes Kepler wanted to find an optimal way to stack cannon balls (of uniform radius) and conjectured a configuration, but could not prove it. This Kepler Conjecture was



(L to R) Henry Cohn (Microsoft Research) in discussion with 2017 SASTRA Prize Winner Maryna Viazovska (EPF Lausanne) and Ken Ono (Emory).

resolved by Thomas Hales in 1998 by combining ingenious geometric optimization arguments with machine calcuations. The sphere packing problem in higher dimensions remained open.

In dimension 8 there is E_8, an exceptional Lie group with a root lattice of rank 8, and in dimension 24 there is the Leech lattice. This gave some hope that the sphere packing problem could be solved in dimensions 8 and 24. Noam Elkies and Henry Cohn made significant progress and conjectured the existence of certain magic auxiliary functions in dimensions 8 and 16, which if determined, would resolve the conjecture in these dimensions. But these magic functions remained elusive. Viazovska produced these functions in dimension 8 by the ingenious use of modular forms. Within the span of a week, working at a furious pace, by extending the ideas in dimension 8, the sphere packing problem in dimension 24 was resolved by Cohn, Kumar, Miller, Radchenko and Viazovska.

Viazovska's modular form techniques are by no means limited to the sphere packing problem. She has discovered something profound that will play a broader role in discrete geometry, analytic number theory, and harmonic analysis.

Maryna Viazovska was born in Kiev, Ukraine, on November 4, 1984. She completed her high school in Kiev in 2001 and her Bachelors in Mathematics in 2005. After completing her Masters at the University of Kaiserslautern in 2007, she joined the University of Bonn where she completed her PhD in 2013 under the direction of Don Zagier. She received the Salem Prize in 2016 and the Clay Research Award in 2017. She received the New Horizons Prize in 2018 along with two other SASTRA Prize winners, Wei Zhang and Zhiwei Yun.

The 2018 Prize: The 2018 SASTRA Ramanujan Prize was jointly awarded to Yifeng Liu (Yale University, USA) and Jack Thorne (Cambridge University, England).

Yifeng Liu had made spectacular contributions to arithmetic geometry and number theory spanning a wide spectrum of topics — arithmetic theta lifts, derivatives of L-functions, the Gan–Gross–Prasad Conjecture, the Beilinson–Bloch–Kato Conjecture, the geometric Langlands program, the p-adic Waldspurger theorem, and the study of étale cohomology of Artin stacks.

His PhD thesis at Columbia University went well beyond the work of Kudla, Rapoport and Yang on L-series connected with the minimal level for Shimura curves over the rationals. Then in three papers pertaining to Bessl and Fourier–Jacobi models, he made significant progress on the Gan–Gross–Prasad Conjecture in the representation theory of classical groups. In collaboration with Shouwu Zhang and Wei Zhang, he studied the p-adic logarithm of Heegner points and proved p-adic versions of theorems of Waldspurger and of Gross–Zagier.

In the 80s, Gross–Zagier and Kolyvagin proved some surprising theorems which implied the celebrated Birch–Swinnerton–Dyer Conjecture for elliptic curves when the analytic rank is either 0 or 1. Liu has successfully extended Kolyvagin type results to higher ranks in the more general framework of the Beilinson–Bloch–Kato Conjecture. Liu has also made inroads into non-Archimedean geometry; he established a non-Archimedean analogue of the famous Calabi Conjecture for abelian varieties over certain p-adic fields.

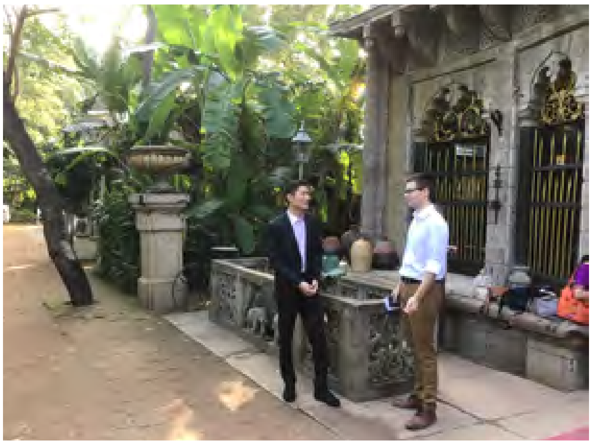
Yifeng Liu was born in Shanghai, China, and received his BS Degree from Peking University in 2007 after which he joined Columbia University, New York, where he received his PhD in 2012 under the direction of Shouwu Zhang. Following that he held the C L E Moore Instructorship at MIT (2012–15) and assistant professorship at Northwestern University (2015–18) before being appointed as an associate professor at Yale University in 2018. He was a recipient of a Sloan Fellowship in 2017.

Jack Thorne had made far reaching contributions to number theory, representation theory, and algebraic geometry. He works in two rather different areas: modularity of Galois representations and arithmetic invariant theory. His outstanding 2012 PhD thesis written at Harvard University was jointly supervised by Professors Richard Taylor and Benedict Gross, two of the dominant figures in contemporary number theory. One outcome of the thesis was his work on arithmetic invariant theory which lead to new results about the sizes of Selmer groups for abelian varieties of small dimension, and bounds for the number of rational and integral points on various types of algebraic curves of genus greater than 1.

Regarding modularity of Galois representations, Thorne has been a central force in eliminating restrictions on the Taylor–Wiles method. Some of the most striking results of Thorne's have appeared in papers with Laurent Clozel. The main input by Thorne is a new method for eliminating the most stubborn restriction in the Taylor–Wiles method, namely the assumption that the residual (mod ℓ) Galois representation under consideration is irreducible.

Thorne's 2015 work with Khare on potential automorphy and the Leopoldt Conjecture introduced a "trick" that plays a key role in an ongoing ten-author collaboration including Thorne, that will establish a potential version of the Shimura–Taniyama Conjecture for elliptic curves over imaginary quadratic fields. Thorne's recent paper establishing that all elliptic curves over Q_∞ are modular is viewed as another major breakthrough.

Jack Thorne received his BA Mathematics degree from Cambridge University in 2007. He then went to Harvard University where he completed his PhD in 2012. Following that he was appointed



Yifeng Liu and Jack Thorne in conversation prior to the award ceremony.

Reader at Cambridge University in 2013, but during the period 2012–17, he was also a Clay Research Fellow of the Clay Mathematics Institute. In 2017 he was awarded the Whitehead Prize of the London Mathematical Society and in 2018 was promoted to full professorship at Cambridge University.

Shaping the development of mathematics: The Fields Medals, with an age limit of 40 for the winners, were instituted with two lofty goals in mind: (i) to recognize pioneering works by brilliant young mathematicians, and (ii) to encourage these young researchers to continue to influence the growth of mathematics. The Fields Medallists have lived up to these great expectations. The SASTRA Ramanujan Prize has a more stringent age limit of 32, and so has recognized brilliant mathematicians even earlier in their careers. It is no exaggeration to say, that the winners of the SASTRA Ramanujan Prizes have also shaped the development of mainstream mathematics and will continue to do so in the years ahead; the fact that the SASTRA laureates have subsequently won major prizes with hallowed traditions, is a testimony to this. Indeed, two of the four 2018 Fields Medalists are former SASTRA Ramanujan Prize Winners — Akshay Venkatesh and Peter Scholze! Similarly, three of the four 2018 New Horizons in Mathematics Prize winners are former SASTRA awardees — Wei Zhang, Zhiwei Yun and Maryna Viazovska.

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Krishnaswami Alladi is professor of mathematics at the University of Florida, where he was department chairman during 1998–2008. He received his PhD from UCLA in 1978. His area of research is number theory. He is the Founder and Editor-in-Chief of The Ramanujan Journal published by Springer. He helped create the SASTRA Ramanujan Prize given to very young mathematicians and has chaired the prize committee since its inception in 2005.

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