Abel Prize 2016 and Fermat's Last Theorem

Y K Leong

The Abel Prize 2016 has been awarded to Sir Andrew Wiles of Oxford University "for his stunning proof of Fermat's Last Theorem by way of the modularity conjecture for semistable elliptic curves, opening a new era in number theory".

The Abel Prize is generally regarded as the mathematics Nobel Prize (the Swedish Nobel Prize Committee does not allocate any prize to mathematics). It was established by the Norwegian Academy of Science and Letters in memory of the Norwegian mathematician Niels Henrik Abel (1802–1829) on the occasion of the bicentenary of his birth. It is funded by the Norwegian government and carries with it a monetary award of 6 million Norwegian krone (about US$700,000). It is awarded annually, the first time in 2003 to the French mathematician Jean-Pierre Serre.

It is interesting to know that more than 100 years ago, a German industrialist Paul Friedrich Wolfskehl (1856–1906), who had an interest in mathematics, bequeathed a sum of 100,000 Deutsch marks (equivalent to one million pounds in 1997 British currency) to be awarded for the solution of Fermat's Last Theorem (FLT). By the time it was finally awarded to Andrew Wiles in 1997, the value of the prize had dwindled to 30,000 pounds because of the hyperinflation Germany suffered after World War I.

Shortly after Wiles corrected his original proof of FLT, major prestigious awards started to roll in. By 1988, he had received 10 of them. That did not stop. In 2005, ten years after the proof, the Shaw Prize rolled in. And now, 20 years after, the Abel Prize arrived. However, the mathematicians' prize, the iconic Fields Medal, was inevitably not among them because of the International Mathematical Union's (IMU) "below 40 years of age" rule for its award (Wiles was 42 years old when he finally proved FLT). But, in an unprecedented move of special recognition, IMU presented him with a Silver Plaque at the Opening Ceremony of the International Mathematical Congress in Berlin in 1998.

Andrew Wiles was at Princeton University between 1982 and 2010, except for a short period from 1988–1990 when he was Royal Society Research Professor at Oxford University. In 2010 he returned to Oxford University's Mathematical Institute where a building (opened in 2013) now bears his name.

The work for which the Abel Prize is awarded this time is of a field (number theory) that is perhaps the oldest and purest area in mathematics. The problem (called "Fermat's Last Theorem") that is solved and mentioned in the prize citation is arguably the most famous problem that has attracted and defied the efforts of many professional and amateur mathematicians for more than 350 years until 1994. It is an unusual story that began in the mid-17th century with a French magistrate Pierre de Fermat (1601–1665) in the town of Toulouse. Fermat was not the run-of-the-mill magistrate. He would spend his leisure time delving into the properties and mysteries of primes and integers. He would communicate his discoveries (often without proof) and issue mathematical challenges to his mathematician friends. The real story began with a statement, casually and almost carelessly, made by Fermat in the margin of a book *Arithmetica* by Diophantus the great Greek number theorist of antiquity: "I have discovered a truly remarkable proof which this margin is too small to contain." The result in question is easy to understand: The equation $x^n + y^n = z^n$ has no positive integer solutions in x, y, z for any integer n greater than 2. (The case when $n = 2$ is, of course, the famous Pythagorean equation $x^2 + y^2 = z^2$ whose integral solutions are well-known.)

This statement has been dubbed "Fermat's Last Theorem" (FLT), conveniently and romantically perhaps but certainly wrongly because it seems that Fermat could only prove it for $n = 4$ and that he did not, in his usual style, mention it as a challenge to others. However, the real story began when Fermat's son Samuel mentioned this statement in his compilation of his father's mathematical work. Not unexpectedly, this set off a spark of interest among mathematicians to prove such a simple and innocuous-looking statement — an interest that may have flickered and wavered but never quite faded during the following centuries. Among those who had tried and only came up with partial success were some of the great mathematicians of the 18th and 19th century, such as Leonhard Euler (1707–1783), Sophie Germain (1776–1831), Carl Friederich Gauss (1777–1855), Adrien-Marie Legendre (1752–1833), Lejeune Dirichlet (1805–1859), Augustin-Louis Cauchy (1789–1857), Gabriel Lamé (1795–1870) and Ernst Kummer (1810–1893) and many others right up to the 20th century.

In 1993, even before Andrew Wiles emerged from a self-imposed period of intense contemplation of FLT in Princeton University to give a series of three lectures at the Isaac Newton Institute for Mathematical Sciences in Cambridge, England, the academic atmosphere was thick with rumour that FLT will be finally proved. When the media caught up with it, the world was abuzz with the sensational news that portrayed a romantic picture (at least to those with a soft spot for mathematical truths) of a scholar emerging from a seven-year period of seclusion with the solution of a 350-year-old problem that has stumped the top mathematical brains for so many centuries. Shortly after the initial elation and euphoria, a mathematician checking through the proof of Wiles found some gaps in it. Needless to say, this had a devastating effect on Wiles who, however, was not one to give up on his efforts of almost a decade. Within a year and with the help of a former student Richard Taylor, he circumvented the difficulties and patched up the gaps in the original version. This resulted in the back-to-back publication of a 109-page paper by Wiles and a 20-page joint paper with Taylor in one single issue of the *Annals of Mathematics* in 1995. They proved the so-called Taniyama–Shimura–Weil conjecture for semistable elliptic curves, which was then used to prove FLT. (This conjecture was basically first presented by Yutaka Taniyama (1927–1958) at the Tokyo-Nikko number theory conference in 1955 and subsequently clarified by Goro Shimura, and later resurrected by André Weil (1906–1998) in 1967. It connects number theory with topology and essentially states that all elliptic curves are modular.)

The crucial mathematical events that led to Wiles' resolution of FLT occurred rapidly in the late 1980s. In 1986, Gerhard Frey had observed and suspected that a counter-example to FLT would give rise to a non-modular elliptic curve, but his work contained a missing link (called the epsilon conjecture) which was identified by Jean-Pierre Serre in 1987. It was only in 1990 that Kenneth Ribet proved the epsilon conjecture, thus paving the way for Wiles. In the wake of Wiles' efforts and building on his work, Christophe Breuil, Brian Conrad, Fred Diamond, and Richard Taylor (the last three being former students of Wiles) proved the full Taniyama–Shimura–Weil conjecture. What began as efforts to prove a simple statement about a diophantine equation gave rise to a new area (algebraic number theory) whose developments led to another new field (algebraic geometry) in mathematics. While the solution of FLT closes one chapter in number theory, it opens up another in modern mathematics.

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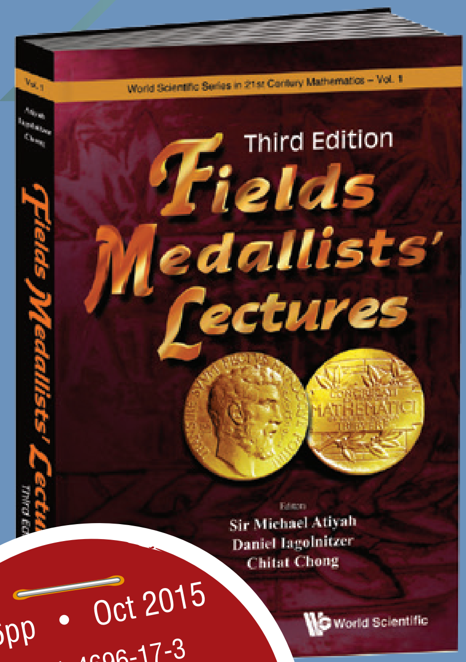

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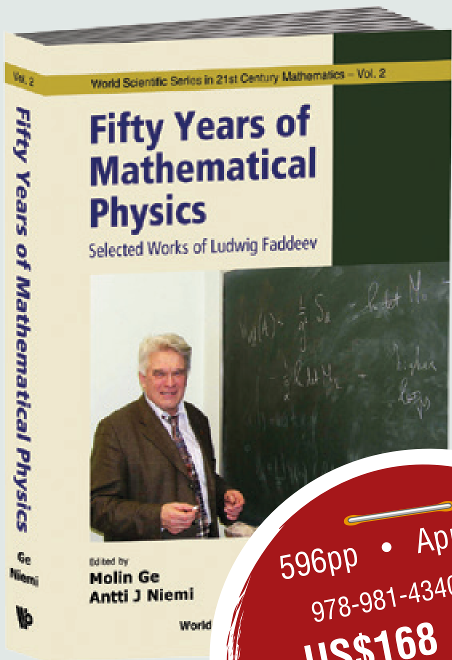

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