Ramaiyengar Sridharan: The Man and His Work

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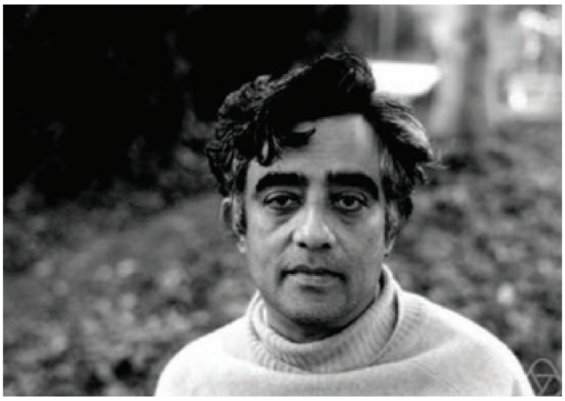

*"Full many a gem of purest ray serene, the dark unfathomed caves of ocean bear
Full many a flower is born to blush unseen, And waste its sweetness on the desert air."
– Thomas Gray, "Elegy written in a country churchyard".*

*"The best portion of good man's life: his nameless, unremembered acts of kindness and love"
– William Wordsworth, Lyrical Ballads.*

This article is a tribute to R Sridharan on the occasion of his eightieth birthday, celebrating his research contributions to mathematics, as well as his role as a teacher and mentor. He was instrumental in leading Indian mathematics on the path of modern algebra in the 1960's. We shall merely attempt to give a chronological overview of his vast and varied contributions, adopting a 'compressing' approach. We hope, nevertheless, that it will encourage readers to dip into various aspects of his work. I was fortunate to be his 'grand student' at the Tata Institute of Fundamental Research (TIFR) (my thesis advisor was Parimala Raman) in the late 1980's, and have been deeply influenced by his teaching skills, his abiding humane nature, empathy and the breadth of his scholarship — not just in math, but also in literature, languages and history.

1. Early Mathematical Years

Ramaiyengar Sridharan was born on July 4, 1935 at Cuddalore in Tamil Nadu. He completed his masters degree (BA, Honors) Vivekananda College Chennai, and joined TIFR in 1955. His early interests in mathematics lay in the areas of algebraic topology, and homotopy theory both of which were emerging, nascent areas. His interests were also kindled by Dowker's lectures at TIFR in 1956–1957.

The turning point at this period in his mathematical life came about in 1956 when Samuel Eilenberg lectured at TIFR. Sridharan solved a problem suggested by Eilenberg on constructing an equivariant version of the so-called canonical map of the 'Nerve' of a covering of topological space. Eilenberg was quick in offering a PhD fellowship under his supervision at Columbia University, which was politely turned down by a diffident young Sridharan. His love for algebra and subsequent immersion in the subject was a consequence of the influences that Eilenberg's lectures had made on him. In the next year, Eilenberg repeated the offer, which Sridharan accepted this time, albeit a little gingerly, being fearful of travelling to such distant shores which were home to a different culture.

2. Columbia Years (1958–1960)

Sridharan departed for Columbia university in September 1958 to begin his PhD studies. An interesting aside that already marked his precociousness was his review of two books that appeared in *Math Student*. One was Father Racine's *Introduction to Abstract Algebra*, [2], and the other was N Jacobson's *Structure of Rings*, [5]. Father Racine played a crucial role in introducing modern mathematics to talented Indian students in Chennai which went a long way in putting India on the world mathematical map in the 1960's.

Being too wise has its odds too as Russell remarked.[a] At Columbia, young Sridharan being depressed wanted to quit mathematics, return to India and take up a teaching job at a local college. However, both Eilenberg and P A Smith, knowing his competency advised him to stick to the path and complete his thesis. Incidentally, having a great interest in varied subjects, he chanced upon Hermann Weyl's remarkable book *Theory of Groups and Quantum Mechanics*.[b] Browsing through it, Sridharan came across the Heisenberg commutation relation '$pq - qp = i\hbar$' and he quickly sensed what lay beneath it, which lead to the inception of his thesis.

Let R be a commutative ring and A an R-algebra. An R-derivation D on A is an R-linear map $D : A \to A$ that satisfies Leibniz rule viz.

$$D(ab) = (Da)b + a(Db), \quad a,\ b \in A.$$

Sridharan proved the following result. Let F be a field of characteristic zero, A an F-algebra on two generators p, q with the relation $pq - qp = 1$. Then every F-derivation D on A is *inner*, i.e. of the form $D(x) = ax - xa$ for some fixed $a \in A$. Eilenberg, very much liking the result told him, "Why don't you generalise this algebra?" Therein lay the genesis of what later got christened as *Sridharan algebras*. He managed to complete his thesis [6, 7], successfully and not only did he defend his thesis in the presence of whole of mathematics faculty of the university including Harish Chandra, but also managed to leave a mark on them. Eilenberg very graciously suggested to Sridharan that he continues his stay at Columbia for a few more months, relax and do more mathematics. However due to the ill health of his father, Sridharan had to return to India on November 1960, cutting short his stay. Thus Columbia's loss was India's gain.

Let us try to delve into his thesis and hope to present a bird's eye view of the same. A *filtered algebra* A over a commutative ring R is an associative, unital R-algebra with an ascending filtration

$$\cdots A_{n-1} \subseteq A_n \subseteq A_{n+1} \cdots$$

by subspaces A_n such that $A_n = (0)$ for negative n, $\cup_{n\in\mathbb{Z}} A_n = A$ and $A_m A_n \subseteq A_{m+n}$ for all m, n.

To any such algebra A, one may functorially associate a graded algebra $S = \mathrm{gr}(A)$, defined by

$$S = \bigoplus_{n\in\mathbb{N}} S_n, \quad \text{where } S_n = A_n/A_{n-1}.$$

Sridharan determined all the filtered, associative algebras A whose associated graded algebra $\mathrm{gr}(A)$ is isomorphic to a symmetric algebra $S(M)$ of a free R-Module M. They are now called *Sridharan algebras* and are constructed as follows. Let $\mathfrak{g}$ be a Lie algebra over F (here F can more generally be a commutative ring) and let $f : \mathfrak{g} \times \mathfrak{g} \to F$ be a 2-cocycle. Define an algebra $U_f(\mathfrak{g}) = T(\mathfrak{g})/I$; here $T(\mathfrak{g})$ is the tensor algebra of $\mathfrak{g}$ over F, and I is the two-sided ideal generated by $x \otimes y - y \otimes x - [x, y] - f(x, y), x, y \in \mathfrak{g}, [x, y]$ being the Lie algebra product.

- If $f = 0$ or a 2-coboundary, this is the usual enveloping algebra $U(\mathfrak{g})$.
- If $\mathfrak{g}$ is a 2-dimensional abelian Lie algebra with basis $\{x, y\}$ and $f(x, y) = 1$ we get the Weyl algebra.
- Under a certain natural filtration, the associated graded algebra is the symmetric algebra on $\mathfrak{g}$.

Sridharan showed that his algebras are characterised by this last property and studied their Hochschild homology.

Example. In dimension 2, $U_f(\mathfrak{g})$ are of the following types:

- $F[x, y]$,
- Weyl algebra A_1 (with generators X, Y and relation $XY - YX = 1$),
- Enveloping algebra of the Lie algebra *Aff*(1), the group of affine transformations of the line; this is an algebra generated by two indeterminates X and Y with the relation $XY - YX = Y$.

Towards the end of his thesis, Sridharan in a

[a] "The whole problem with the world is that fools and fanatics are always so certain of themselves, and wise people so full of doubts."

[b] Originally "*Gruppentheorie und Quantenmechanik*" in German. It took Weyl to fruitfully realise the group and representation theories within the domain of Quantum mechanics.

short remark,[c] hinted that such an aforementioned generalisation would have some role to play in quantum mechanics and it did!

Subsequently, Sridharan's study of Hochschild homology led to the conclusion that not only can the Weyl algebras be realised as algebras of differential operators on some field, but also, as stated in the Example, as a Sridharan enveloping algebra of 2-dimensional Lie algebras. This can also be interpreted as Poincaré Lemma for quantum differential forms and plays an important role in modern theoretical physics.

3. Return to TIFR

After returning to TIFR, Sridharan began his mathematical life as a thesis advisor almost immediately, supervising the doctoral work of Amit Roy, Shrikant Mahadeo Bhatwadekar, Bongale Chawathe, etc. This also marked the beginning of his close association with the Centre for Mathematics at Bombay University, thereby unconsciously paving the path of modern algebra which would later flourish in different departments in India. It was at this time, in the early 1960's that Hyman Bass visited TIFR and gave his famous lectures on algebraic K-theory. Sridharan, who at that time was lecturing on Projective geometry at the Bombay University, thought of studying projective geometry over arbitrary commutative rings. He also studied cancellation problem for Azumaya Algebra. In 1967, Sridharan visited ETH, Zurich and with Ojanguren, he proved the fundamental theorem of projective geometry for commutative rings [17], which we describe below.

Let A, B be commutative rings, and M a free A-module. The projective space $P(M)$ is defined to be the set of all A-free direct summands of M of rank 1. Suppose M (resp. N) is a free A-module (resp. B-module). A projectivity is a map $\alpha : P(M) \to P(N)$ which is bijective and has the following property:

- For all $p_1, p_2, p_3 \in P(M)$, we have $\alpha p_1 \subset \alpha p_2 + \alpha p_3$ if and only if $p_1 \subset p_2 + p_3$.

If $\sigma : A \to B$ is a ring isomorphism, then any σ-semilinear isomorphism $\Phi : M \to N$ induces a projectivity $P(\Phi) : P(M) \to P(N)$. With this backdrop, we now state the theorem of Ojanguren and Sridharan.

Theorem 1. *Suppose rank M and rank N are both greater than or equal to 3. If $\alpha : P(M) \to P(N)$ is a projectivity, then there is an isomorphism $\sigma : A \to B$ and a σ-semilinear isomorphism $\Phi : M \to N$ such that $\alpha = P(\Phi)$. Further, σ is unique and Φ is unique up to a multiplicative constant.*

We now turn to another theorem of Ojanguren and Sridharan on 'Cancellation for Azumaya algebras', [19]. Recall that Azumaya algebras over rings are forms of matrix algebras over the ring. Simply put, if R is a commutative ring, $\mathcal{A}/R$ is an Azumaya algebra of degree n if and only if there exists a faithfully flat extension R' of R such that $\mathcal{A} \otimes_R R' \simeq M_n(R')$, the ring of $n \times n$ matrices over R'.

The question of cancellation for Azumaya algebras is the following. Suppose $\mathcal{A}$ and $\mathcal{B}$ are two Azumaya algebras over a commutative ring R. Suppose there exists an Azumaya algebra C/R such that $\mathcal{A} \otimes_R C \simeq \mathcal{B} \otimes_R C$. Does this imply that $\mathcal{A} \simeq \mathcal{B}$? Equivalently, if $M_n(\mathcal{A}) \simeq M_n(\mathcal{B})$, do we then have $\mathcal{A} \simeq \mathcal{B}$?

This is known to be true when R is the polynomial ring $F[X]$ over a field F. Ojanguren and Sridharan showed that it is false when R is the polynomial ring $F[X, Y]$ in two variables and F is the centre of a finite dimensional noncommutative division ring D. More precisely, they consider a division ring D, and elements α, β in D such that $\alpha\beta \neq \beta\alpha$. They then consider the $D[X, Y]$-linear map $\phi : (D[X, Y])^2 \to D[X, Y]$ defined by $\phi(1, 0) = X + \alpha, \phi(0, 1) = Y + \beta$. This map is onto and $\mathrm{Ker}(\phi)$ is a stably free projective right module of rank one over the noncommutative ring $D[X, Y]$. They prove that $I := \mathrm{Ker}(\phi)$ is not free, and that, if $\mathcal{A} = D[X, Y]$ and $\mathcal{B} = \mathrm{End}_{\mathcal{A}}(I)$, then $M_3(\mathcal{A}) \simeq M_3(\mathcal{B})$ but $\mathcal{A}$ is not isomorphic to $\mathcal{B}$.

In fact they exhibit a necessary and sufficient condition for cancellation, which we now outline. Suppose that we have an isomorphism $M_n(A) \simeq M_n(B)$ for two rings A and B. Then this implies an isomorphism $A \simeq B$ inducing the isomorphism between the matrix rings, if and only if every right projective A-module P such that $P^n \simeq A^n$, is an A-bimodule.

Another beautiful area of joint work with Ojanguren is that of Galois theory of purely inseparable

[c]Some of the groundbreaking discoveries sprung out from such humble footnotes. To quote an instance, Max Born left a humble footnote in his magnum opus 1926 paper, that wave function represents probability and such a probability is given by amplitude squared $|\psi|^2$, which changed the way we look at quantum mechanics thereafter. Born, M. (1926). "Zur Quantenmechanik der Stossvorgänge". Zeitschrift für Physik 37 (12): 863–867.

extensions, [18]. Suppose that F is a field of prime characteristic p, and L is a purely inseparable extension such that $L^p \subset F$. Then Ojanguren and Sridharan showed that there exists a Galois correspondence between subextensions of L/F and the so-called restricted L-subspaces of the space of F-derivations $\mathrm{Der}_F(L)$ which are closed with respect to the Point topology of $\mathrm{Der}_F(L)$. A subspace D is *restricted* if it is a vector space over L which is closed under taking pth powers.

They also prove that any restricted L-subspace of $\mathrm{Der}_F L$ is also closed under the Lie bracket. This work extends results of Jacobson and corrects an earlier result of Gerstenhaber.

4. The 1970s and the 80s

It was but natural to wander from such questions in to 'Serre's problem'. Serre's problem is the question whether projective modules over polynomial rings $F[X_1, X_2, \ldots, X_n]$ for a field F are actually free. In 1957, Seshadri showed that the answer is yes when $n = 2$. Serre's problem is closely related to assertions on being able to 'cancel' projective modules, as one explores the relationship between projective modules being stably free and actually free.[d] This striking and deep question was eventually settled (affirmatively and independently) by Quillen and Suslin. It was also at this time that there was considerable advancement in building an algebraic theory of quadratic forms, over fields, rings and eventually over algebraic varieties. Over varieties, the objects of study were quadratic spaces, which basically consisted of a vector bundle on the variety supporting a suitably generalised notion of bilinear form. A natural question that arose in this context when the underlying variety was the affine n-space $\mathbb{A}^n_F$ over F was the following: when are quadratic spaces over such affine spaces extended from quadratic forms over the base field F? This can be viewed as a natural analogue of Serre's problem in the context of quadratic spaces over affine spaces. Parimala joined TIFR in the 1970's and Sridharan suggested a problem motivated by his joint work with Ojanguren along these lines. Jointly with Sridharan, [22, 23], Parimala studied this question when one replaced F by a division ring, in particular the division algebra $\mathbb{H}$ of real quaternions. We denote the set of projective ideals (under a certain equivalence relation) of $\mathbb{H}[X, Y]$ by $\mathcal{P}$. Parimala classified the elements of $\mathcal{P}$ in terms of Hermitian spaces (vector bundles with a Hermitian structure) of rank two and trivial determinant over $\mathbb{C}[X, Y]$. Restricting scalars from the field of complex numbers to the reals, one then obtains anisotropic quadratic spaces of rank 4 over $\mathbb{R}[X, Y]$, which are indecomposable and hence not extended from $\mathbb{R}$. Thus an element $P \in \mathcal{P}$ naturally gives rise to an anisotropic quadratic space of rank 4 over $\mathbb{R}[X, Y]$, which we denote by $\alpha_{\mathcal{P}}$. In joint work with Knus and Parimala, [26], Sridharan showed that extending $\alpha_{\mathcal{P}}$ to $\mathbb{P}^2(\mathbb{R})$ would produce indecomposable rank 4 quadratic spaces over $\mathbb{P}^2(\mathbb{R})$. This was in contrast to work of Harder who had shown that all anisotropic quadratic spaces over the real projective line $\mathbb{P}^1(\mathbb{R})$ are extended from $\mathbb{R}$.

Theorem 2. *(Knus–Parimala–Sridharan). Let F be a field of characteristic different from 2. Any quadratic space of $\mathbb{A}^2(F)$ extends to $\mathbb{P}^2(F)$, and this extension is unique if quadratic Space is anisotropic.*

They also proved a similar result for Hermitian forms. We remark that this result is related to Barth's classification of stable, rank 2 vector bundles of type $(0, 2)$, viz. bundles with first Chern class c_1 equal to zero, and second Chern class c_2 equal to 2, over $\mathbb{P}^2(\mathbb{C})$. This work was later used in a joint paper, [29], of Ojanguren, Parimala and Sridharan to explicitly construct quadratic spaces of rank $4n, n \geq 1$, over $\mathbb{R}[X, Y]$. Such quadratic spaces, in particular, are not extended from the field of reals.

5. The 1980s and the 90s

This period marked a golden age of vigorous activity in the area of quadratic forms at TIFR. In the 1960's and the 1970's, after the work of E Witt, and the work of A Pfister and M Knebusch, the algebraic theory of quadratic forms blossomed into an area with interesting connections to Milnor's algebraic K-theory and Galois cohomology, culminating in the famous Milnor conjecture. We remark that this conjecture was solved by V Voevodsky in the middle of the 1990's, which won him a Fields Medal. Knebusch had defined the Witt ring $W(X)$, for any scheme X, generalising the Witt ring $W(F)$ of quadratic forms over a Field F. It was only natural that Sridharan and Parimala made an easy transition to the study of Witt groups of

[d] Aforementioned work of Sridharan and Ojanguren shows that its false in general if F is a noncommutative division ring.

varieties after their earlier results on quadratic and Hermitian bundles.

Simultaneously, Sridharan, Parimala and Knus studied the discriminants of involutions on central simple algebras. Let F be a field of characteristic different from 2, and let A/F be a central simple algebra. An *involution* σ of the first kind on A is an F-linear anti-automorphism of order 2. If $A = \mathrm{End}_F(V)$, then every involution of the first kind is the adjoint involution with respect to some non-singular bilinear form b on V, which is either symmetric or skew-symmetric. The involution σ on A is called *orthogonal* (respectively *symplectic*) if, over the algebraic closure of F, it becomes the adjoint involution with respect to some non-singular bilinear form on V which is symmetric (respectively skew-symmetric). Moreover the discriminant of the bilinear form, characterising the involution over the algebraic closure, descends to an F-invariant of the involution, the *discriminant* of the involution. In joint work with Knus and Parimala, [41] Sridharan showed that the discriminant of σ can also be directly defined as the square class of the reduced norm of any invertible skew-symmetric element. They showed in [40] that an involution σ on a biquaternion algebra decomposes as tensor product of two involutions on quaternion algebras if and only if the discriminant of σ is trivial.

Suresh joined TIFR in 1989 and became part of the quadratic forms school, completing his PhD under Parimala. If the degree of the central simple algebra A is greater than or equal to 4, then Parimala, Sridharan and Suresh showed [43] that the set of discriminants of orthogonal involutions on A is the full group of square classes of reduced norms of invertible elements on A. We remark that it was unknown, prior to this, whether the algebra A even carried an orthogonal involution of trivial discriminant!

Parimala, Sridharan and Suresh [48] studied the Witt groups of algebras with involutions and constructed exact sequences of these groups. This later provided a key inductive step in the resolution of Conjecture II of Serre by Bayer–Parimala. In a joint work [54], Ojanguren–Parimala–Sridharan–Suresh also proved purity for the Witt ring of a 3-dimensional regular local ring. All these results were used later in understanding quadratic forms, involutions and Witt rings of algebraic varieties. Another noteworthy result proved by Parimala, Sridharan and Suresh [60] was a generalisation of a classical theorem of Springer. Suppose that F is a field of characteristic not 2 and let q be a quadratic form over F which is anisotropic. Springer's theorem asserts that q then remains anisotropic over any odd degree extension L of k under the natural map. In particular, this gives an injective map of Witt rings $W(F) \to W(L)$. Parimala, Sridharan and Suresh generalise this result for Hermitian forms. Let A be a quaternion algebra over a field F (char $F \neq 2$) and let h be a Hermitian form over A relative to an involution of the first kind. Then if h is anisotropic over F, it remains anisotropic over any odd degree extension of F.

As the quadratic forms school in TIFR continued to attract students, Sridharan played an active role in mentoring them, helping them master the requisite background to embark on a research problem. We briefly mention his work on the Maslov index with Preeti Raman, [57], classification of Albert algebras, as well as results on Jordan algebras and $\mathbb{F}_4$-bundles with Maneesh Thakur and Parimala, [52, 55]. Preeti and Maneesh both graduated from TIFR.

6. Mentor for the New Millennium

As the new millennium dawned, Sridharan had to bid a graceful adieu to his long association with TIFR. True to his innate scholarly calling, he had no hesitation in moving to Chennai Mathematical Institute (CMI) to continue with his teaching and research. It would be difficult to overstate the role he played in nurturing students at CMI who were interested in algebra and number theory. He also made deep contributions to his other pet passion, History, especially the history of mathematics, and in particular ancient Indian mathematics. Being a polyglot, his expertise in Sanskrit, Tamil, German, and contemporary literature has been widely reflected in his articles and lectures. Due to his competency in sanskrit language, the production crew of the only existing Sanskrit movie on the life of Adi Shankara (directed by G V Iyer) approached him to help them with English subtitles which he successfully did. Despite having to face challenges on the health front, Sridharan continued to do the things he believed in and loved most, namely teaching, mentoring and scholarly research. His beautiful work on enumerative combinatorics in prosody, [69], the works of Piṅgala, [69, 84], elliptic functions, [63, 64, 77], to mention just a few, demonstrate the breadth of his interests and depth of his research. I cannot pretend to elaborate on the work done in the last fifteen years and I am glad that there are others more competent than me, who have written about this. I was fortunate to spend around a year at CMI in 2007–2008 and witness first hand, the energy and scholarly ambience he brought to CMI.

I would like to close this article by insisting that any write-up on Sridharan is bound to appear incomplete without reference to his nature as a human being. His sensitivity, empathy and deeply humane qualities have provided comfort and solace to many, especially to young students. I was personally a beneficiary and it left an indelible memory in me.

He was also very encouraging and a genuine supporter of women at the workplace, and a true believer in the natural capabilities of women in all spheres of life. I have learnt many valuable lessons on being a gentle and genuine human being from Sridharan. His other appealing qualities include a self-deprecating sense of humour and a disarming frankness. Of course, people close to him can recount his almost 'obsessive' appreciation of Wodehouse novels and Jeeves, immortalised in a newspaper column, [38], of his. Another aspect that should be stressed about Sridharan is his quest for perfection. He would never tire of seeking ways to improve whatever it was that he was doing and instilled this trait in his students as well. Those who have sat through Sridharan's lectures also know how he works hard at communicating difficult mathematics to the audience, a skill which he again tried to impart to his students.

தாமின் புறுவது உலகின் புறக் கண்டு காமுறுவர் கற்றறிந் தார்
"The learned foster learning more, On seeing the world enjoy their lore."

உவப்பத் தலைக்கூடி உள்ளப் பிரிதல் அனைத்தே புலவர் தொழில்.
"To meet with joy and part with thought, Of learned men this is the art."

- Thirukkural, Thiruvallvar.

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