Yum-Tong Siu: Hongkong-Princeton-Harvard, A Path of Several Complex Variables

This issue features an interview with Yum-Tong Siu of Harvard University, who made fundamental contributions to complex, algebraic and analytic geometry, and complex differential geometry. It was originally published in Issue 26 (July–December 2015) of the newsletter Imprints of the Institute for Mathematical Sciences (IMS), National University of Singapore. The following is an excerpt of the interview article reprinted with the kind permission of IMS.

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Siu was born in Guangzhou in the midst of World War II, and his father moved the family to Macau and later, Hong Kong, in the aftermath of the communist takeover of the mainland. He received a classical Chinese education from Pui Ching Primary School and Pui Ching Middle School in Macau. It was this early exposure to classical education that sparked his interest in Chinese literature, philosophy, and history, an interest that has turned into a life-long passion. Later he transferred to Pui Ching Middle School in Hong Kong. Every day before taking a ferry to go from Hong Kong Island to the school in Kowloon on the other side of the harbour, he took an early walk with his father, a textile merchant. It was during these walks and over dim sum in a restaurant that his curious father would ask him to share information about academic subjects and current events. Shortly before Siu's graduation from high school, he almost had to transfer to a public school with lower tuition because of setbacks in his father's business. Only a timely reduction in tuition fees granted by Pui Ching enabled him to continue and to graduate from high school. (Pui Ching Middle school has an impressive record of students who subsequently went on to achieve excellence in engineering, mathematics and the natural sciences.) Though financially unable to go to the United States for further studies, Siu won a government scholarship to study at the University of Hong Kong, where he excelled in competitive swimming, obtained a BA in mathematics and met his wife-to-be. After which, in a somewhat unexpected way, he went from the University of Minnesota (for a Master's degree) to Princeton University (for a PhD). He taught briefly at Purdue and Notre Dame before moving on to and rising quickly on the academic ladder at Yale University and Stanford University. In 1982 he went to Harvard University and became the William Elwood Byerly Professor ten years later.

In a long and distinguished career, Siu was for a quarter of a century a world-renowned leading figure in complex analysis, having (according to the citation for his award of the Bergman Prize in 1993) "settled a long and impressive list of problems and opened new directions of research through highly imaginative and original use of sheaf theory, partial differential equations and differential geometry". The earlier part of his work (in the theory of several complex variables) is on the theory of extension of coherent subsheaves (joint work with G Trautman) and coherent sheaves, the structure and extension of closed positive currents and the extension of meromorphic maps across subvarieties.

His later research is conducted at the interface of complex analysis, differential geometry and algebraic geometry and has resolved numerous outstanding problems and conjecture 33 among others, his work on geometric strong rigidity, his joint work with N Mok and S K Yeung on super-rigidity, Frankel conjecture (with S T Yau, Fields Medal 1982), effective results in algebraic geometry such as the freeness part of the Fujita conjecture (with U Angehrn) and the effective Matsusaka big theorem, the conjecture of the deformational invariance of plurigenera, and the conjecture on the hyperbolicity of generic hypersurfaces of sufficiently high degree in complex projective space.

In addition to a large research output, he has written several monographs on the topics of his research interest. He has served on the editorial boards of *Annals of Mathematics* and *Journal of Differential Geometry*. He has been invited to many universities throughout the world and was invited to give talks at the International Congress of Mathematicians three times, two of which were plenary lectures. His awards include the Bergman Prize, Guggenheim Fellowship, Sloan Fellowship and honorary doctorates from the University of Hong Kong, University of Bochum (Germany) and University of Macau. He is a member of the American Academy of Arts and Sciences, US National Academy of Sciences, Academia Sinica (Taiwan), Corresponding Member of the Göttingen Academy of Sciences and Foreign Member of the Chinese Academy of Sciences. He is actively involved in scientific committees to promote and maintain professional awareness in mathematics. In particular, he has travelled frequently to various countries in Asia, contributing to their efforts in raising international interaction. He was on the Advisory Committee for the Shaw Prize (established by Hong Kong philanthropist Run Run Shaw) in Mathematical Sciences. He was Chair of the National Committee for Mathematics (National Research Council, National Academy of Science). He was a member of the Scientific Advisory Board of the Clay Mathematics Institute. Currently he is on the Scientific Advisory Board for the Institute for Mathematics Sciences (IMS), National University of Singapore (NUS) and for the Institute of Advanced Studies, Nanyang Technological University (NTU), Singapore.

Imprints: You went from BA (Hong Kong University or HKU) to PhD (Princeton University). Please tell us how you came to take this path?

Yum-Tong Siu: At the University of Hong Kong I majored in mathematics. It was offered inside the Faculty of Arts.

I: Isn't it offered in the Faculty of Science?

S: Yes, in those days, mathematics was offered in both the Faculty of Science and Faculty of Arts. In the former, mathematics must be combined with another science subject such as physics and chemistry. It was only in the Faculty of Arts that one could major in mathematics alone. In the old British system, a major in just one subject meant taking 9 papers in the final examination. I took 9 papers in mathematics. At that time I was thinking of going to Germany for further study. As preparation, I took German in my first year. The instructor was the cultural attaché from the German consulate in Hong Kong. The class was very small as not too many HKU students were interested in German. The main job of the instructor was to recruit Hong Kong students to eventually study in Germany. After graduating from HKU, I was offered a German Academic Exchange Service or DAAD scholarship. (DAAD stands for *Deutscher Akademischer Austauschdienst*). However, my dream of studying in Germany was thwarted. When I contacted the German university that I was interested in, I found out that the HKU system and German university system were not compatible. I had to start my study in Germany from the level of the *Arbitur* which is the high school graduation exam in Germany. If I thought I already had the education up to the level of their *Vordiplom*, I was allowed to take the exams right away. But I figured I wouldn't pass right away their *Vordiplom* exam and it made no sense to me to start from the level of high school graduation all over again. I thought I should study in the United States but I had no application form. By that time, it was rather late in terms of applying to graduate study elsewhere. I am talking about pre-computer days. All correspondence with overseas universities had to be carried out by snail mail, not email or even fax. Fortunately, my HKU classmate Tsit Yuen Lam had several application forms left. He had already been accepted by Columbia University. (Lam is now a professor emeritus in the Department of Mathematics at UC Berkeley.) I took the application forms to show my professor (Professor Yung-Chow Wong) and asked for his advice. He recommended the University of Minnesota because Eugenio Calabi was there. So I applied and was accepted. I went to Minnesota, studied with Calabi, and earned a Master's degree before deciding to leave. After that year, Calabi left Minnesota and went to University of Pennsylvania. I think he stayed there till he retired. Another reason for my not wanting to continue with my PhD study in Minnesota was the brutal winter weather there.

I: Why didn't you go to Pennsylvania?

S: While at the University of Minnesota, I actually had trouble understanding Calabi's lectures. I was his only student in class; the rest were professors and postdocs. I was not really the targeted audience but then outside of class he helped me tremendously. Whenever I asked him questions privately after class, he would explain a lot of things to me. I really learned a lot from him; he was extremely knowledgeable. Still, I felt that his style was very different from what I was used to in Hong Kong. He had a really panoramic view, which certainly helped me conceptually, but he often skipped technical details. At that time I felt a need for more foundational content, so I decided to go to Princeton instead of following Calabi to Pennsylvania. The Princeton style was also different. There were no basic courses in Princeton and the professors would only lecture about their own work, sort of seminar style. Students of comparable background would form small study groups in order to learn the basic material together. I worked with three fellow students.

I: How did you come to choose functions of several complex variables as the area of research for your PhD?

S: When I went to Princeton, besides student seminars, I also attended a seminar run by Bochner [Saloman Bochner (1899–1982)]. Gunning [Robert Clifford Gunning] was also in the seminar. I also took a course with Gunning. I found the subject very interesting and Gunning was also a very good and encouraging teacher. He had many students at that time. Having also learned from the other students of his, gradually I was drawn more into the field of several complex variables. During my stay at Princeton (1964–1966) the field was blessed with many breakthrough activities, for example, exciting results obtained by Grauert [Hans Grauert (1930–2011)] and others in Germany and France. At Princeton, Gunning was lecturing from material in the book *Analytic Functions of Several Complex Variables* that he wrote with Rossi [Hugo Rossi]. I found the subject fascinating. The material available in book form was presented well. My fellow students were helpful. It was in this academic atmosphere that I chose several complex variables as my research interest.

I: Were you the only student from Asia at that time?

S: I was not the only Asian mathematics student at Princeton. The other student was K Y Lam [Kee Yuen Lam], the older brother of T Y Lam. K Y Lam, a student of Professor Steenrod [Norman Earl Steenrod (1910–1971)], was a couple of years ahead of me in HKU. Whereas T Y Lam is in algebra, K Y Lam is in topology. Yes, there were only two Chinese mathematics graduate students at the time, but Wu-Yi Hsiang, a visitor to the Institute of Advanced Study at Princeton, would show up in seminars of Princeton University. Yiu-Hung Chan and Hung-Hsi Wu were also around at the Institute for Advanced Study. Both Hung-Hsi Wu and Yiu-Hung Chan were from my high school, four years ahead of me. Yiu-Hung Chan later quit mathematics or maybe just quit research in mathematics. He went back to Hong Kong to teach but was not active in research anymore. Hung-Hsi Wu later became Professor of Mathematics at UC Berkeley.

I: He's in differential geometry, I think.

S: Yes, he's now retired from Berkeley.

I: He's very keen in education.

S: Oh, you know him well.

I: In fact, I interviewed him two years ago. He's now almost totally engaged in training teachers.

S: Yes, he has always been interested in teaching and then got more and more involved in teaching, trying to reach out to those at the fundamental level, like even primary school pupils.

I: You have been on the faculty of Harvard University for the past 30 years or so. What is it that makes you so attached to Harvard?

S: Well, Harvard has both excellent students and very distinguished faculty members. It's thus a very exciting place. I first taught at Purdue for one year and then three years at Notre Dame and then I taught at Yale for eight years. Actually I left Yale because of the weather. There was a big snow storm, blizzard of '78, so I left in '78 to go to Stanford. Weather was not a consideration, however, when I went to Harvard. A main reason for my departure from Stanford was my wife's career. She finished her doctorate in social work (Columbia University) in 1982 and had difficulty finding social work faculty position at or near Stanford. I received offers from Harvard, MIT and Princeton. The greater Boston area with its over 50 colleges would provide more job opportunities for my wife. I chose to accept Harvard's offer. After a year, my wife did find a faculty position at Wheelock College of Boston. She taught there for 22 years and was Chair of the Department of Social Work for ten years. She chose to retire in 2005. Well, we have lived in the Boston area for over 30 years. I think we have made the right decision, not only professionally for me but for my family as well. Harvard students are extremely good because Harvard remains a popular choice for the most talented students. For example, freshmen enrolled in Math 55 (a special course for the most talented students) are a pleasure to teach. The main part of the course consists of challenging weekly assignments. I taught this course a couple of times and found my task to be time-consuming but highly rewarding.

I: I remember reading some comments from the blog of some of your students that they find it to be a tough course.

S: I think it's not only mine. That particular Harvard mathematics course is meant to be tough. The choice of material for the course is to serve two purposes. One is to challenge students. Another is not to discourage them. One seeks a balance between the two. I believe that gearing the course content to the level of the students is true of all courses.

I: Probably your expectations were higher.

S: Actually it depends on the class composition of a given year. Math 55 is a completely dynamic situation. In this only-one-of-its-kind course, you sort of look at what the students in the class have accomplished, and based on feedback from students, you then channel the students to more and more difficult material, with the goal of providing maximum challenge without discouraging them. To me, it is a good thing if students find the course to be challenging. After all, it is supposed to be tough! Student opinions about the course can be short-term (immediately after finishing the course) and long-term as they reflect on the learning experience much later. I know most Math 55 students, despite the hard work involved, enjoyed both the learning outcome and the process. This unique course has become an important tool for Harvard to recruit gifted students who aspire to be mathematicians.

I: I believe that you have settled a number of outstanding conjectures in analysis and algebraic geometry. What is the style of your research? Do you pick specific difficult problems to work on or do you try and relate and synthesize different concepts and theories?

S: For research it's hard to fully know beforehand the nature of the problem and likelihood of successfully solving it. It's not that I pick this problem, I want to solve it because, at least in my own case, my work depends so much on what other people have done in the past. My view of my research is that it is akin to putting together a jigsaw puzzle. Sometimes you see already quite a number of pieces in place and you think you have some way of putting in more. Whether I pursue a research topic depends on what is interesting, what is available, and what is feasible at that time. It's just like investment. You do several things at the same time. There is no telling which one would bear fruit. So when I work on things, it depends on what is interesting but it also depends on my background. At the inception of my career, I started out in differential geometry. As I told you, I went to Minnesota and learnt from Calabi. Later when I went to Princeton, at that time the work of Grothendieck [Alexander Grothendieck (1928–2014), Fields Medal 1966] was fashionable, so I learnt algebraic geometry, all the language and so forth. Later I picked up several complex variables which interface with several areas, including the methods of algebraic geometry and methods of PDE (partial differential equations). At that time the method of the so-called complex Neumann problem or $\bar{\partial}$-estimates became available, as did methods of integral kernels and methods of global differential geometry. Thus, I worked at the interface of these areas and on quite a number of conjectures. I was lucky enough to solve some. Of course, we are all aware what the prevailing problems are. I simply choose the ones that I feel are feasible at that point, given my background and the available methods. I leave behind other problems on which I cannot launch a meaningful attack. That's my way of approach to research. I suspect that probably most mathematicians do it the same way. In the interface or bridge between algebra and analysis, I usually use differential geometric, algebraic geometric and also PDE methods involving hard analysis.

I: Have you ever tried to do applications of what you have done to other fields?

S: You mean, that would affect the livelihood of people?

I: I mean, physics or . . .

S: Yes, actually some of the problems were originally proposed by people in other areas. But I actually never go directly to other areas to see how these things are being used. Now with globalisation not only geographically but also in terms of subject matter, research has become more interdisciplinary, resulting in more interactions among researchers. There are people who serve as bridges at various stages, some closer to mathematics, some closer to other fields. Many of the mathematical developments have been spurred by other areas. Some of my own work has been motivated by other fields. The field of differential equations certainly consists of practical aspects, and in geometry now physics plays a greater role.

I: A question about undergraduate curriculum. In the undergraduate mathematics curriculum, a course on real analysis (functions of a real variable) is usually followed by one on real multivariate calculus (functions of several real variables) whereas a first course on complex analysis is very rarely followed by one on functions of several complex variables. Is this due to pedagogical reasons or deeper mathematical reasons?

S: Usually there are two kinds of undergraduate curriculum. The first is of a service nature, that is, a curriculum serving people in other fields. For various reasons, some students in other fields need to learn mathematics at an undergraduate level. The curriculum varies according to the kind of mathematics they need. Sometimes they need only the results without the theory, sometimes a combination of theory and results. The other curriculum is for our own mathematics students. We are using this kind of curriculum to train a new generation of mathematicians who will replace us at some point in the future.

In the service curriculum, the main purpose is to help other fields, so we just do whatever people in other disciplines want us to provide. Although we certainly could provide suggestions and feedback to the other departments about course content, our primary role is to serve them. In the other situation where we are grooming future mathematicians, the curriculum content beyond the basics depends very much on faculty members of a particular mathematics department. After mastering foundation materials, second-stage mathematics majors are steered towards courses and seminars offered by existing faculty members. The curriculum very much depends on the composition of the department and the professional bias of faculty members. When I teach, of course, I would like students to understand what I consider interesting, so I try to give more information about several complex variables. Such knowledge of several complex variables would help students gain a different perspective of one complex variable.

After so many years of teaching, I have concluded that it's important to achieve a good balance between abstract rigorous reasoning and concrete mathematical structures such as computational formulas. Students exposed to an overemphasis on the abstract side may only know how to make logical arguments but lack enough knowledge about structure, actual formulas, and specific examples. Interestingly, as I become older, I tend to favour knowledge about examples of actual structures or formulas. I believe that's what mathematics is. Of course, it's always good to know the general theory but then the theory is, to start with, built up from specific examples. Being able to work with specific examples would lead to a true understanding of what is really going on and increase the probability for fruitful research.

I: Results in complex dynamics, such as fractals, often give rise to and are conversely suggested by computer simulation. Has the computer played any role in the theory of functions of several complex variables?

S: Yes, as you said, in the case of complex dynamics, the computer is of great benefit. It helps in performing computations. In the past, we did long computations by hand, which was not only time-consuming but also prone to making mistakes. Computers in a way are more reliable provided that the programme is correctly written. From the computer, one can quickly obtain a lot more examples, which are presented well visually, sometimes with colours. These examples may provide motivation to go forth in one's research. But there is a caveat. Even with the use of computers, in the final analysis, you still have to get back to pure logical reasoning – mathematically rigorous reasoning.

I: Did you use the computer for your research?

S: I did it a number of times for checking computations, but I trust my own logical reasoning more. At one time when I computed curvature, especially with the situation of symmetric Riemannian manifolds, I did use the computer to help me guess what the final result is likely to look like. But then eventually I still had to do it by hand to be completely satisfied that there were no errors. One reason is my lack of trust in my own ability to program and, more importantly, to debug the program.

I: You have spent many years lecturing and sharing your experiences with mathematicians in countries like China, Taiwan, Hong Kong, South Korea and Singapore among others. What are some of your most memorable experiences of your visits in Asia?

S: First, let me say that recently there has been a proliferation of activity in terms of conferences and special programmes in all these countries you mentioned. This development is very exciting, providing a lot of opportunities for young people. One difference between young people here and the United States and Europe is that Asians are generally more hard working. It's just the way they are brought up or the way the education system works, I think. In some ways, it is more satisfying for me to work with Asians because of their work ethics. They are simply more willing to put in effort. On the other hand, Asians out of their respect for elders may tend to discuss mathematics less directly. Perhaps, this impression is due to my usually brief stays in Asia. Even though I have made many visits, the duration of each visit is not long enough to interact in depth with young mathematicians. Or perhaps the age difference has put some distance between them and me. The interactions between them and their local senior mathematicians may well be different, resulting in a different level of discussion.

I: Maybe they are differential towards their elders.

S: Yes, I believe that may be one of the reasons. Also it's changing. It's good to see a really international mix. You also see here in the workshops and programmes a lot of mathematicians from Europe and North America. Eventually everything is going to even out. Besides, there are more Asian students, postdocs and mathematicians visiting Europe and North America. With the kind of funding Asian countries are putting into mathematical research, the rate of increase in research will certainly far outpace what is currently going on in Europe and North America.

I: Now that the last ICM [International Congress of Mathematicians] 2010 was held in India and the coming ICM 2014 will be held in South Korea, what are your expectations of the next phase of mathematical development in Asia, in general, and East Asia, in particular?

S: Oh, I think the pace will pick up faster and faster. You also see the number of institutes and meeting venues springing up everywhere in Asia. And globalisation is really accelerating. Besides the examples you just cited of the ICM in India, there was also the ICM 2002 in China. There is simply more and more activity at the international level in Asia. I believe that globalisation will make the whole world more homogeneous so far as mathematical research activity is concerned. Such a development is not only inevitable but desirable as well.

I: Do you think that at the regional level mathematical research in Asia-Pacific will be comparable to that in Europe?

S: Of course, Europe has a really long history. It takes time to develop a mathematical tradition and provide role models to students. For example, in China for a long time, the development was more in geometry and analysis because there were a number of senior people there. Now I see many young people in algebra and algebraic geometry because of a whole new generation of mathematicians in that area. So when it comes to Asia I can see that not only is the level rising and picking up in pace but also the diversification of subjects. Eventually I think there will not be much difference. At this time, because of historical reasons, there is still a difference. In the near future, hopefully not too far away, research output and quality will be more or less the same between Asia and Europe and North America.

I: Can I be a bit more specific? I think Hong Kong has produced a number of very distinguished mathematicians during your generation.

S: Actually from my own high school I could count quite a number of people. For example, Hung-Hsi Wu, myself, S T Yau [Shing Tung Yau], S Y Cheng [Shiu-Yuen Cheng], Lawrence Ein, Kai Yuen Hu (who was on the faculty here and retired a number of years ago), Pit-Mann Wong, Bun Wong. What is unique about Pui Ching Middle School in Hong Kong is that it is not only private but also uses Chinese as a medium of instruction, unlike the mainstream Anglo public and private schools in Hong Kong. I came to Hong Kong after the communists took over and established the People's Republic. Many in the intellectual community in China also relocated to Hong Kong. Their academic degrees might have been earned in China and in countries which are not part of the British Commonwealth (England, Canada and Australia). As a result they were considered not qualified by the British government in Hong Kong to teach in Hong Kong. Those who could not find desirable teaching jobs in the Anglo schools ended up teaching in Pui Ching and 26 similar schools. As a private school, Pui Ching did not have to meet the hiring criteria set up by the government. I consider it a blessing to have had very good teachers, who not only imparted their wealth of knowledge but also their worldview. High school students are usually quite easily influenced by good teachers. I was one of them. My teachers attempted to motivate us in an interesting way by using as role models former Pui Ching students who have gone to study in the United States. Because Pui Ching school was not part of the mainstream, its graduates usually could not directly go to the University of Hong Kong for their college education. I and a couple of my classmates were exceptions. Some of my classmates went to the United States right after high school graduation, eventually ending up in institutions like Caltech and UC Berkeley. The teachers always cited Pui Ching alumni as examples for current students to follow, pointing out their areas of study, namely physics, mathematics and engineering. A lot of my classmates actually ended up as engineers.

In those days, the winning of the Nobel Prize by Lee Tsung-Dao and Yang Chen-Ning was a big event. Also the results and developments in physics were very exciting to me and my peers. I remember clearly the incident of Tsien Hsue-Shen [Qian Xuesen (1911–2009), aerospace scientist, "Father of Chinese rocketry"]. He went back to China and he was allowed to leave the United States in exchange for some airmen who were held captive by the North Koreans after their planes were shot down. When I was young, I read news about this exchange and saw photos of Tsien Hsue-Shen and his family (two children and his wife) crossing the Lo Wu Bridge as the airmen walked across the bridge from China into Hong Kong. Students in my high school in those days were all thinking about mathematical sciences, engineering and similar fields. In contrast, the mainstream schools in Hong Kong produced students for the University of Hong Kong who, upon graduation became educational officers, administrative officers and medical officers to help the British govern Hong Kong. Their career paths were totally different from the kind of career paths which I and my high school classmates took.

I: It's rather strange because you would expect them to go to England, right?

S: Yes, but then we were not in the mainstream because we were in a private school that used Chinese for instruction. We had no particular identification with the British! Students from Taiwan also went to the United States but only for graduate school because at that time they had to finish their military service first after completing their undergraduate degrees. So the phenomenon I described represents a unique situation within a very specific timeframe and political context.

I: Many, if not most of, the programmes of IMS during the first 10 years of its operation, tend to be of an applied nature or closely related to applied fields. (The previous director Louis Chen is a probabilist.) Now that both the present director (Chong Chi Tat) and the new SAB Chairman (namely yourself) are "pure mathematicians", will there be a slight change in emphasis, if not direction, in the offering of IMS programmes in the future?

S: I don't think so. Roger Howe, the chairman before me and my former Yale colleague, is a pure mathematician. The then director Louis, of course, is a probabilist. However, if you look at the current advisory board, you can see that most are applied or related to applied areas. Douglas Arnold (from Minnesota) was the president of SIAM and certainly would be applied. Then you have Fan Jianqing (he's professor of finance), very applied. Then Wolfgang Hackbusch from Max Planck Institute in Germany, he's also more in PDE, numerical analysis. Of course, there's Hugh Woodin in logic, and myself in pure mathematics. Louis Chen still sits on the board, as does Chong Chi Tat. I think it's a good balance and a continuation of what has been working. I succeeded Roger Howe as Chair of SAB. I assume that I am Chair because I'm so much older than the others. The field is not the primary consideration when selecting the SAB chair; age probably is. In summary, I don't think there's going to be any change in the direction of IMS. We always look for the most exciting areas in mathematical sciences, whether pure or applied probability, PDE, numerical mathematics, finance, computing and so forth.

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