Interview with Tony Rothman

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Sujatha Ramdorai: Tell us a little about yourself.

Tony Rothman: I'm a theoretical physicist who has specialised in general relativity and cosmology. Most of my research has concerned the very early universe and black holes. I am just finishing a six-year appointment in the physics department at Princeton University and will be teaching at The College of New Jersey in the fall. I've also done a fair amount of writing for the general public (sometimes I hesitate to call it "popular"). Quite recently I published my tenth book, *Firebird*, which is a novel set in a fusion-research laboratory. I think it is unusual in that it isn't science fiction, but an attempt to base a novel on real science. To the best of my abilities the science is totally accurate and, unfortunately, the politics too. I've also just drafted a play about the famous sixteenth-century Cardano–Tartaglia feud over the cubic equation. It's been fun, but difficult. One question has been how "accurate" to make it. Much of what is written about it in the semi-popular literature is nothing more than fairy tales. Several recent books have Tartaglia causing Cardano's arrest for heresy — 13 years after Tartaglia died! At any rate, it's something that I needed to get done.

SR: What appealed to you in the Sangaku story and Hidetoshi's work that you decided to collaborate on the book?

TR: While in high school my favourite math subject was certainly geometry. I think mathematicians have either algebraic imaginations or geometric imaginations. I don't consider myself a terribly creative mathematician — like most physicists I use mathematics to solve problems — but my own imagination is certainly geometric. I suspect this is one reason I fell in love with relativity, which is a very geometric subject. My contact with Sangaku came about during a very specific space-time event. One day, as I recall, in winter 1989–1990, I stopped by Freeman Dyson's office at the Institute for Advanced Study in Princeton. We were probably just planning to have lunch. As soon as Freeman raised his hand to say hello, he said, "Take a look at this," and handed me the Sangaku-problem book Hidetoshi had just published with Dan Pedoe, who had been Freeman's math teacher long ago in England. I had absolutely no idea of what "Sangaku" or "temple geometry" meant. As I leafed through the book with dropped jaw, Freeman stood there laughing. I found the problems visually striking, quite different from anything I had seen in school — they even looked Japanese. But the main impression was how damn difficult they were. For all my love of geometry, I quickly realised to my embarrassment that I hadn't the faintest idea of how to solve most of them. The fact that they were found in temples and had evidently been largely created by farmers and peasants was an additional embarrassment, not to mention extremely intriguing. I bought a copy of the book for myself, worked on some of the problems and eventually contacted Hidetoshi about a possible *Scientific American* article, which I wrote with his assistance. The article sat at the magazine for three years before it was published — even though I had been an editor there. When it finally appeared, it proved fairly influential — I think it was the first major piece in the West about Sangaku — and it helped make temple geometry part of the world heritage of mathematics.

So I am happy about that. I hadn't initially intended to write an entire book about temple geometry. I prefer to write fiction, actually, but it has become almost impossible to get fiction published. And so, around 2005, when all my fiction projects had collapsed, I contacted Princeton University Press about doing a book on Sangaku and the editor there, Vickie Kearn, quickly agreed (literally within about twenty minutes, although it was actually the second time I approached them; a previous editor had expected me to pay for it!) I did all this without even telling Hidetoshi, but he was quite glad to hear the news.

SR: One of the things that strikes the reader — and which you comment yourself — is the aesthetic side of the Sangaku. What are your thoughts on this.

TR: As I said, I found the problems quite beautiful — miniature Japanese works of art. Like most scientists, I suspect, I am drawn to "clean" artwork — I am always struck by how many of my colleagues have collections of African art, and I do as well. Japanese art certainly fits the bill. In fact, I think the Japanese are incapable of creating anything ugly. I was also struck by the asymmetry in many of the problems, compared to the Greek-inspired problems we all face in high school. From Daisetz Suzuki's books on Zen, it seems that asymmetry is a characteristic feature of Japanese art. So I suspect that the whole aesthetic of Sangaku evolved from the Japanese artistic aesthetic. I'm certain that many of the problems evolved from everyday situations and objects — like fans, which are really sectors of circles, and origami designs. The tablets themselves are also beautiful, brightly coloured, and in one case, surrounded by a striking dragon frame.

SR: Can you tell us more about the whole collaborative process?

TR: It wasn't easy. To this day, Hidetoshi and I have never met. I don't speak any Japanese and his English, although it's improved over the years, is far from his native language. The whole thing was done by email. Over the two years we worked on it, I'd guess we exchanged about a thousand emails. Luckily, mathematical terminology is limited and, usually, well defined, but sometimes we would exchange ten emails just to clarify one sentence. Hidetoshi is the expert on Sangaku; my role was basically editorial. He would send me the raw material and I would check it for errors and rewrite the problem statements into respectable English. The intro chapters I wrote pretty much from scratch. Also, I wanted the book to appeal to non-mathematicians, so I tried to avoid technical terms when possible, even when they might have made things clearer to geometers. We had some organisational issues as well. Hidetoshi wanted to organise the book by tablet, but this resulted in very easy problems being placed side by side with nearly impossible problems, and I felt that this would discourage a lot of readers, not to mention make presentation of solutions extremely difficult. So I reordered everything, placing easy problems first and harder problems later. Most of the solutions were either traditional or Hidetoshi's, but I also contributed a few and did all the line drawings, mainly because Hidetoshi's drawing software wasn't compatible with anything I had. The whole thing ended up being a gigantic jigsaw puzzle. From a design perspective, it was certainly the most complicated book I've worked on. The Princeton University Press art director, Dimitri Karetnikov, was very helpful in this regard.

SR: Has working on this project led you to explore other areas of "Japanese Science" or other forms of Japanese knowledge or culture, especially from that period or earlier?

TR: I am not a scholar of Japanese culture — and don't speak Japanese — so I haven't plunged far into related areas, but I am intrigued by certain aspects of Japanese mathematical history, which seem to me not well understood. For instance, the feudal Japanese mathematicians didn't know calculus — at least what we regard as calculus — and we don't know anything about how they handled differentiation. Yet some Sangaku problems seem to require differential calculus for their solution. It is a mystery to me how the Japanese solved them. Also, there is a whole field known as "Rangaku", literally "Dutch Learning", which concerns foreign knowledge that seeped into Japan during the period of national isolation through the Dutch trading post on Deshima island. There seems to be quite a debate among scholars about just what the Japanese knew of foreign science and when they knew it, but the Wikipedia article on Rangaku, for example, isn't very satisfactory. I would be interested in learning more about Japanese knowledge of foreign developments in mathematics. Finally, my *Scientific American* article and the book seem to have given people the impression that everybody in Edo-period Japan was creating Sangaku. It is difficult to estimate the number of original tablets, but even if there were

50,000, that would only be about 150 a year over three centuries, and that is surely an upper limit. Since many of the problems on different Sangaku are duplicates, I suspect that we have most of the problems that were created — only several thousand. So it may not have been such a widespread cultural phenomenon. One or two other Sangaku investigators have written to me about such matters and perhaps they should all get together and try to sort them out.

SR: The book is well publicised in the West. What have the reactions been to what one might call a "Coffee Table" book on mathematics?

TR: It was my intent from the start to create exactly the first coffee-table math book; I even called it that and I guess it has been received as such, although I don't know of too many people who bought it just to look at the pictures. (Given the available resources, Princeton University Press did a good job. The book would have been even more beautiful if Abrams had published it, but it would have cost $100.) Most of the feedback I've gotten has been from mathematics faculty members at various universities who are planning a trip to Japan and would like to see a Sangaku in the flesh. I pass them on to Hidetoshi, who knows where they all are. (I should say that most of the original Sangaku have been removed from the temples and are either in museums or in temple storehouses, where you need prior permission to view them.) One interesting outcome is that an artist in Santa Fe, Jean Constant, has based a whole series of his and his students' works on Sangaku. They're quite striking. A furniture maker has also created a "sangaku" line. I'm always glad when science or math inspires artists, even if their creations are metaphorical. Nevertheless, in this case most of my mail has come from mathematics people.

SR: Tell us a little about your experience in this whole transcultural, mathematical journey that straddles two civilisations.

TR: I think I've already given some idea about that. In general I believe in culture shock — it keeps you on your toes. Certainly, to interact with someone from a different culture whom you've never met — especially by email — takes a lot of patience. The whole situation is a minefield for misunderstanding, and sometimes I think Hidetoshi and I blew each other up. When I taught in Korea a few years ago and would hang out with the students, the long silences made me uncomfortable, until they told me that silence was admired in their culture. Luckily, mathematics itself is universal. The problems were basically Euclidean geometry problems, and although the Japanese often attacked them with methods that wouldn't have occurred to me, I was nevertheless able to understand what they were doing. I do feel that I wasn't the ideal person to collaborate with on the book. A mathematician fluent in Japanese and versed in Japanese history would have been a better choice. My only qualification was that I stepped up to the plate.

SR: When did you actually first see a Sangaku in reality?

TR: Believe it or not, I never have. I've travelled widely around the world, and have lived for many years abroad, but for some reason have never been to Japan. As I just said, I wasn't the ideal person to do this book. I hope someday to get there and then Hidetoshi will show me some.

SR: Did you notice anything different in the way the problems were posed and answered, compared to your own training in the West?

TR: Sure. The repeated, intricate use of the Pythagorean theorem was really ingenious, if at times cumbersome. It's amazing how much you can do with just the Pythagorean theorem. One eye-opener was the Japanese way of dealing with ellipses, which is quite different from ours. The Japanese mathematicians viewed an ellipse as a slice through a right circular cylinder, not as a conic section. A circle inscribed in the ellipse was the projection of a sphere in the cylinder onto the slice. They could then use the Pythagorean theorem to connect the various important lengths involved. This "3-D" approach allows you to solve some really difficult problems, which I never would have been able to do using the usual equation for an ellipse. In fact, you don't even need the usual equation for an ellipse.

I did find some of the problems ill-posed. Hidetoshi tells me this is a feature of traditional Japanese mathematics. For instance, take problem 7.12 in the book, which is an unsolved problem in which you are asked to find the radius of three identical circles, two of which are inscribed in an ellipse, which is itself inscribed in a right triangle along with the third circle. It wasn't at all clear to me at first that there even was an analytic solution. Recently Jesu Alvarez Lobo from Spain has sent me his solution. I haven't worked through it, but it seems to be a tour-de-force, over thirty pages (arXiv:1110.1299). He does find an analytic, closed solution for an isosceles right triangle, but for the general case he can only get an implicit solution. Some of the traditional solutions are from another galaxy. We are often taught to draw auxiliary lines in high school geometry courses to solve problems. In at least one Sangaku problem (6.3), the author of the solution introduces an entire auxiliary circle, which at first seems to have nothing to do with the problem whatsoever. I doubt I'd ever have thought of that solution. And again, it wasn't entirely obvious to me that there even was one. Lobo showed that the solution exists only for a particular base angle of the isoceles triangle involved in the problem. Also, many of the solutions, especially by Yoshida Tameyuki, assume rather sophisticated lemmas or other steps, which aren't stated. I don't know whether Yoshida just assumed everyone knew them or what, but I doubt I'd ever present a proof with so many important details missing. One thing I learnt, is that to solve these intricate Sangaku problems, you have to make a *really* good drawing, not just a sketch. And in proving things about triangles you should never draw a 45-degree line.

SR: There were algebraic, arithmetic as well as geometric problems that were posed, though geometry seems to have been the most popular. Some of the problems, as you observe in the book have appeared in other guises in other cultures. Why do you think Geometry was more popular?

TR: Many problems, both algebraic and geometric, pop up in different cultures, just as most scientific discoveries are made multiple times. I don't know whether the duplication of the math problems was due to cross-fertilisation by "word of mouth" over the centuries, or whether they cropped up independently. Certainly every time I do something in physics, no matter how obscure it seems, somebody else always claims to have done it first! If geometry problems have been more popular, it must be because of the visual appeal, and in some sense geometry is easier than algebra. Even if you are algebraically challenged, as I often am, you can often solve problems geometrically, and many of the basic geometric theorems regarding angles and so forth are pretty self-evident, so you don't really have to prove them before solving a problem. In writing this play about Tartaglia and Cardano, I was looking at the Tartaglia–Cardano solution to the depressed cubic equation. They did this before modern algebraic notation existed and so you might think it really hard. But to find the cubic formula is actually really easy if you think geometrically and remember that a cubic equation must give the volume of a cube. If you slice up the cube as those fellows did, the cubic formula falls out almost immediately. It's a good lesson. Nowadays we have algebratised geometry to such an extent that we've often make things more complicated than they really are.

SR: "Sacred Math", "Temple Geometry".... In the West it would almost be anachronistic to juxtapose science and the spiritual in this manner, yet this appears natural in the East. Especially the thought of the problems and solutions being offerings to the divine! What are your thoughts on this....

TR: The idea that mathematical tablets were presented as religious offerings is very appealing to me, but of course, the Buddhist idea of God, or the divine, is much more abstract than Western ideas. Some readers of our book have suggested that our use of the word "God" in the translation of some of the tablet inscriptions may not be accurate. Unfortunately, since I don't know Japanese, I am unable to say. Russians often speak of "lighting a candle to God", whether they are talking about creating a work of art or solving a scientific problem. That's more or less how I feel about it, although in my case "God" may be even vaguer than it is for the average Buddhist. I think, though, that Sangaku served several purposes. Since hanging tablets in temples and shrines was a long-established tradition in Japan before the advent of Sangaku, one can't rule out the possibility that many people were just doing it for "fun", or maybe with about as much religious fervour as most Americans celebrate Christmas. Some tablets were apparently created by classes at small schools, called *Jyuku*, and almost certainly these were hung in the temples as advertisements for the school. In any case, since the Sangaku almost always contain an answer, but rarely the solution, they were pretty clearly issued as challenges to other "worshippers".

SR: Any lessons you have learnt or any other related thoughts you might want to share?

TR: It's become clear to me that geometry education has declined considerably, at least in the US. To write *Sacred Mathematics*, I consulted a number of textbooks that are currently used in high schools, but they were totally inadequate to solve most Sangaku problems. To write the chapter on inversion, I needed to go back to texts that were literally a century old, e.g. Clement Durrell's geometry. Inversion simply isn't taught anymore, except in some advanced college math courses. And it's not even that hard!

SR: Thanks very much for taking the time for the interview.

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