Competent but Anxious

Audrey Wang

Although Taiwanese students are known for being good at math, they lack interest and confidence in the subject.

To the outside observer, Taiwanese students exhibit a remarkable aptitude for math. The 2011 Trends in International Mathematics and Science Study (TIMSS), for example, a worldwide survey conducted by the International Association for the Evaluation of Educational Achievement every four years, ranked the math ability of Taiwan's fourth-graders number 4 among the 52 countries and regions surveyed and Taiwan's eighth-graders number 3 among 45 countries and regions.

Those a bit more attuned to Taiwan's education system, however, have long known that as local students move on through junior high and senior high school, they begin to feel a pervasive "math anxiety". In fact, that uneasiness is revealed by another part of the TIMSS survey, which evaluates how much students like math and how much confidence they have in studying the subject. Surprisingly, the same fourth-grade and eighth-grade students in Taiwan scored among the lowest in both categories, indicating that while the education system turns out elementary and junior high school students with good math competence, they find the process painful and discouraging.

Lai You-tang（賴友堂）is instruction director of the math teaching program at the Humanistic Education Foundation (HEF), a nonprofit organisation established in 1987 and dedicated to promoting educational reform in Taiwan. Lai sums up the strange paradox of learning math in Taiwan. "The [math] performance of Taiwanese students has always been among the best. In general, our children's basic calculation ability is far better than that of their counterparts in the United States and [many] European countries," Lai says. "But while they perform quite well, they feel terrible about their ability. It means they may have acquired their ability in a not-so-healthy learning environment."

Education authorities have long searched for ways to make the process of learning math less traumatic. Before 1975, Taiwan's elementary schools taught mostly arithmetic, or basic numerical calculation, and focused little on the wider field of mathematics itself, which is defined as the study of numbers, equations, functions and geometric shapes and the relationships between them. The first stirrings of reform began that year when mathematics was officially introduced as a subject. Accompanying that move was the release of a standardised curriculum that aimed to teach children the math skills needed for everyday life while also developing their learning strategies and ability to reason. Under the new curriculum, elementary school students still learned the four arithmetic operations, but they also took on basic geometry as well as certain formulas and equations. Most learning continued to take place through memorisation and repetitive application, however, as that approach was believed to be the quickest way to help students master calculation skills.

Lai believes the 1975 curriculum had merit, but also great room for improvement. On one hand, he says, its courses played an important role in equipping citizens with necessary life skills, especially during the time when Taiwan's economy was going through a fast expansion. "Nowadays, most people in Taiwan can perform simple, basic operations without much difficulty," he explains. "In this aspect, the teaching materials and guidelines passed down from [1975] can be considered effective."

On the other hand, Lai points out that although the 1975 reform nominally targeted the development of students' learning strategies and reasoning ability, in practice it still placed too much emphasis on teaching arithmetic skills. Students with poorly developed reasoning ability, he explains, begin to struggle in math after they enter middle school or high school, where math becomes a much larger, much more abstract subject. "Math therefore becomes a great fear for a lot of those students," he notes.

Partly in response to such concerns, in 1996 a larger reform effort was launched that implemented a new-generation curriculum known as constructive mathematics in Taiwan's elementary schools. That radical change is believed to have been triggered by the 1989 call by the National Council of Teachers of Mathematics in the United States for a new curriculum guided by a constructivist understanding of how students learn. While traditional math teaching methods focus on rote memorisation and practice, the constructivist approach maintains that students must develop their own skills

by understanding relevant mathematics concepts and background first. "The key to the philosophy is having the students 'construct' the knowledge by themselves," explains Jen Wei-yang（任維勇）, a math teacher at Taipei First Girls' High School. "To prevent students from thinking there's only one way to solve a problem, you're not supposed to tell students the established rules directly."

Many math teachers responded to the new guidelines by discouraging students from memorising any rules, including the multiplication table. Also common was an insistence that students use horizontal calculations instead of vertical. For example, when asked to multiply 3 times 7, children were encouraged to figure out the answer by writing 3 + 3 + 3 + 3 + 3 + 3 + 3 = 21. To calculate 25 plus 32, students were taught to show their train of thought by writing 25 + 32 = 20 + 5 + 30 + 2 = 20 + 30 + 5 + 2 = 50 + 5 + 2 = 50 + 7 = 57.

Children who failed to follow the horizontal approach and answered the question by relying on memorisation of the multiplication table or mental calculations would often see their score reduced even though they got the correct answer. Jen says many parents considered constructive math a nightmare as they found the new approach too inefficient and could not see how it related to the computational rules they had learned.

Children in a HEF math thinking class work out the concept of place values for numbers (Photo Courtesy of Humanistic Education Foundation).

Lih Ko-wei（李國偉）is a research fellow in the Institute of Mathematics at Academia Sinica, Taiwan's foremost research institution, as well as the convener of the math education section under the National Science Council's Department of Science Education. Lih says that on the whole, the constructive approach is a good way to teach math. What ultimately undermined the 1996 reform effort, he says, was that many instructors did not understand the proper methods for teaching constructive math and therefore turned what should have been an inquiry-based learning system into just another set of rules and techniques to be learned by rote.

Along similar lines, Lai says, while instructors were encouraged to employ innovation and creativity in teaching the new math, that approach was unfortunately watered down by the time it reached elementary schools. The initial promoters of constructive math — two professors at National Taiwan University in Taipei — cultivated several "seed teachers", and the seed teachers spread the approach to teacher representatives in each school district, who in turn taught the method to math teachers in schools, Lai explains. "The core values of [constructive mathematics] were lost after being passed down so many times," he says, adding that as the teaching method was transferred from teacher to teacher, many rules that greatly deviated from the true constructivist approach were created and reinforced.

Instructors tasked with teaching constructive math were also greatly hampered by the lack of quality teaching materials, Lai says. For example, there were not nearly enough "why" questions listed in textbooks for teachers to use for initiating class discussions, he says. Thus, when teachers ran out of such open-ended questions for the students to consider, they reverted to teaching computational rules, he says. As a result, the turn toward constructive math failed to address the problem of rote learning.

After middle school teachers began noticing a general decline in the arithmetic ability of students who studied constructive mathematics, a new curriculum was put into place in 2005. According to Lai, the latest curriculum is seen as a compromise between the 1975 and 1996 curricula, as it does not emphasise use of constructivist teaching methods but still encourages instructors to teach math concepts in a diverse and creative manner.

In Taiwan, one of the most controversial aspects of constructive math was its insistence that understanding come before memorisation. The 2005 curriculum,

however, accepts that memorisation can be useful in some cases. Reflecting the move back toward a balanced approach, Lih says that he finds nothing wrong in asking children to memorise certain rules and formulas when they begin to learn mathematics. "Beginning when we're little, we acquire a lot of so-called procedural knowledge," he says, referring to knowledge gained through performing a task. "It's like learning to ride a bike — you don't know how the bike works, but you gradually figure it out by actually riding it. Take memorising the times table as an example — it's not a big burden for children. By memorising it, they gain familiarity with multiplication, which makes the learning process easier in the future."

One of the most influential factors behind the continued reliance on memorisation in mathematics teaching is the extreme emphasis placed on academic performance in Taiwan. "For most teachers I know, helping students get good grades is the most important goal because it's what the parents want and expect from school teachers," Jen says. "Only after that goal is achieved do teachers try to make the courses a little fun so that the subject will not seem too boring for students."

Lih agrees, saying that the most significant characteristic of math education in Taiwan is the heavy pressure to get the good grades that enable one to enter a top school. In such a competitive environment, according to the researcher, teachers prefer to give students as much practice as possible to ensure high test scores, leaving instructors little opportunity to experiment with creative teaching methods.

Reliance on Memorisation

Such heavy reliance on memorisation, as some educators and parents see it, is a major cause of students' math anxiety, as the drills can enhance students' ability to calculate, but not their understanding of how the formulas work or why particular calculations should be used. Lai says, for example, when asked to estimate the area of Antarctica, many local children know enough to draw a circle around the continent and calculate the area of the circle by using the formula $A=nr^2$, but the majority of them do not know why they should use a circle to find the area, nor why the formula uses a circle's radius, as represented by r, instead of diameter. Without a good understanding of the formula itself, students easily become anxious about whether they have memorised or used it correctly, Lai explains.

While it is unlikely that further curriculum reforms specifically targeting rote memorisation will appear in the near future, a larger effort is being made to address the intense grade pressure students face. For many years, achieving a high score on Taiwan's senior high school entrance examination has been seen as the key to gaining admission to a good high school and from there on to a good university and good job. That will change in 2014, however, with the implementation of a new 12-year education policy that greatly reduces the emphasis on senior high entrance exam scores. Lih and other educators are hopeful that the policy will lead to a decrease in grade pressure, which could give teachers more freedom to spend time imparting the background knowledge that helps students understand why a certain kind of math should be used in the first place.

Students raise questions during a HEF math thinking class aimed at helping them look into the "whys" of math (Photo Courtesy of Humanistic Education Foundation).

Another institutional problem that leads to math anxiety is that math textbooks used by Taiwan's elementary and middle schools cover a lot more topics than those used in many other countries, Lai says. Almost inevitably, learning more topics means that young students are required to memorise many more rules. The problem, Lai says, is that while children appear to be accumulating a massive amount of knowledge, their learning is so shallow that they lack confidence in their ability to use it, as the TIMSS report shows.

"Fragmented knowledge is taught to help students cope with tests," Lih says, but adds that learning such bits and pieces does little to build genuine understanding. "It's interesting that sometimes students can manage to answer a question correctly without understanding the concept behind it," he says.

Instead of teaching students a small amount about many topics, Lai maintains it is better to cover fewer topics while providing comprehensive background information about each. In this case, the impetus for reform comes not from the government, but from private organisations like the HEF, which launched its own math program around 10 years ago. The foundation

began by publishing its own math textbooks and in 2008 branched out into offering regular instruction in what it calls "math thinking" courses, which are aimed at encouraging in-depth learning. "For example, we go deeper into math subjects by illustrating them with stories and pictures and encouraging discussion," Lai says. More than 2,000 students have taken HEF's math classes each year since regular courses were launched.

The "Whys" of Learning

Math thinking also attempts to impart deeper understanding by reviving aspects of the constructivist approach to teaching math. "The most important thing is to teach children to look into the 'whys' of learning math," Lai says. For example, when children learn to tell time, the conventional first step is to teach them that the shorter hand on the clock shows the hour and the longer hand indicates the minute. However, the math thinking program does not teach such rules, but asks children to figure out why the clock hands are designed as they are, Lai says. After some discussion and well-timed prodding by the teacher, the young students eventually figure out that the minute hand needs to be long enough to come close to the minute dial, thereby clearly distinguishing the minute, he adds.

Despite the high scores Taiwan's students receive on math tests, educators remain focused on reducing math anxiety because gaining competence in the subject is closely linked to improved reasoning ability. "Math helps people develop the habit of logical thinking and cultivate their ability to make a rational analysis and judgment in everyday life," Jen says. Lai concurs, saying "People may ask why it's useful to learn the square root of three and square root of five, but what really matters is not these numbers; instead, it's the reasoning we develop during the process of understanding such concepts. Logical thinking isn't just needed by those in the high-tech industry — it's needed by everyone in every part of their daily routine."

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