Fighting Asian Catastrophes with Mathematics

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Abstract. The Asian continent is particularly exposed to catastrophes, both natural and man-made. When disaster strikes, event accounts tend to be descriptive and phenomenological, leaving open the deeper questions of understanding suited to mathematical enquiry. Not only are these questions of intrinsic mathematical interest, but their solution contributes to mitigating the devastating economic and human loss wrought by Asian catastrophes.

As is apparent from the country's very own name, in the Netherlands, flooding is an ever present threat. When an eminent Dutch mathematician, Laurens de Haan, wrote a paper titled, "Fighting the Arch-Enemy with Mathematics", it was evident to its people that the subject was extreme flood risk, which has historically been a lethal scourge of the Netherlands. If a dike is overtopped by a coastal storm surge, the consequences can be calamitous when the land inside the dike is below sea-level. Regardless of the height of a dyke, there is always a chance, however slim it may be, that it may be overtopped. The designated annual tolerance is set very low accordingly: 1/10,000. From the statistics of coastal water heights measured, it is possible to estimate appropriate dike design levels using extreme value distributions. De Haan was a prolific contributor to extreme value theory, in particular, to distributions F in the domain of attraction of an extreme value distribution G, where sequences $a_n > 0$ and b_n $(n \geq 1)$ exist, such that $F^n(a_n x + b_n) \to G(x)$.

Fig. 1. On 1 February 1953, a great North Sea storm surge killed almost 2000 people in the Netherlands. After this tragedy, mathematicians were brought in to assist in calculating dike heights. [Photo from BBC News archive]

Although the Netherlands is intrinsically flood-prone, the Dutch are thankful to be less vulnerable to other forms of geological hazards; windstorms are typically more of a threat to agriculture than to people. However, for Asians, earthquakes, volcanic eruptions, tsunamis, landslides and typhoons may be as severe a personal threat as a flood. Indeed, for Dutch researchers analysing Earth hazards, their primary scientific laboratory has been Indonesia, formerly a Dutch colony known as the Dutch East Indies. If the Netherlands is a key example of extreme flood risk, Indonesia forms an even more important example of extreme geological risk. The greatest historical volcanic eruption was the eruption of Tambora in 1815, which discharged 150 cubic kilometres of ash into the atmosphere. Scientific understanding of Nature starts with observations such as this; but why should we anticipate cataclysmic events on such an enormous scale?

It took a mathematician, Benoit Mandelbrot, to find the answer. For any empirical observation to be deeply understood, there needs to be the discovery of a mathematical representation. Nature has its own geometry, which is not composed of the regular shapes familiar from Euclid. Mandelbrot observed that "clouds are not spheres, mountains are not cones, and lightning does not travel in a straight line". Furthermore, there is a continuum of distinct length scales of patterns: a photograph of a small rock may look similar to a photograph of a cliff face. Discovering the self-similar geometry of coastlines from a British scientist, Lewis Fry Richardson, Mandelbrot developed fractal geometry into a powerful tool for understanding the natural world, and for comprehending the fundamental logarithmic scale of natural hazards.

If natural catastrophes are pathological events punctuating the relative tranquillity of geodynamic processes, it is because the underlying geometry of Nature is also pathological, in a classical Euclidean sense. Taking the simple example of a coastline, measured in intervals of length ε, approximation by a broken line requires a number of intervals proportional to ε^{-D}, where the exponent D depends on the jaggedness of the coast.

The fractal geometry of fault movement is central to the size distribution of earthquakes. The magnitude of an earthquake is correlated to the length of rupture, which varies self-similarly from metres to hundreds of kilometres. Across the Indian Ocean, from Sri Lanka to Thailand, fatalities of the Sumatra tsunami of December 26, 2004 fell victim to the fractal geometry of Nature. For most Asian coastlines threatened by tsunamis, there are no engineered tsunami barriers to prevent the ingress of tsunami waves. The word "tsunami" is Japanese for harbour wave, and the Japanese archipelago is especially prone to tsunamis. Tsunami barriers have been constructed in Japan, but the design basis has traditionally been deterministic, as once it was for dikes in the Netherlands. In the absence of any risk-based criterion, the worst historical event may be taken as the reference for design, possibly with a modest increment as a safety margin to reach some notional upper plausible bound.

The Japanese tsunami of March 11, 2011 was a catastrophe of gigantic proportions, greatly exacerbated by the under-design of the tsunami protection around the Fukushima nuclear power plant. Civil engineers are predominantly responsible for safety construction standards, but their education has traditionally focused on deterministic principles of Newtonian mechanics, with comparatively little time for a basic training in stochastic processes. Following the practical example of Laurens de Haan, mathematicians are needed to assist engineers in coming to terms with quantifying the uncertainties in the threat environment, so as to improve the design of safety-critical infrastructure.

The magnitude of the earthquake that generated the tsunami was 9.0, which was a record for the region, and higher than the maximum hitherto anticipated by Japanese seismologists from geological arguments, which was 8.3. The mathematical theory of records is of intrinsic interest in itself, but also can illuminate event sequence patterns. Suppose that the following event sequence is observed $X_1, X_2, X_3\ldots$, and that the maxima are $M_1, M_2\ldots$. The elegant mathematical structure of such record sequences has been explored, and affords insight into maximum magnitudes, where the regional earthquake catalogue is sufficiently extensive.

Once a tsunami is generated by an earthquake of large magnitude, the ocean propagation of the tsunami and its run-up on land are analysable by applied mathematicians, using the principles of classical hydrodynamics. However, the task of forecasting accurately the run-up heights along an irregular coastline, allowing for spatial variability in bathymetry and topography, is a significant numerical modelling challenge. Solution of the nonlinear shallow water equations itself can lead to some idealised run-up formulae, expressible succinctly and elegantly in terms of Bessel function integrals, but the greater spatial complexity of actual run-ups is captured on videos of destructive tsunamis. Where tsunami barriers are absent or deficient, the timing of a call for evacuation is crucial. Natives of islands vulnerable to the occurances of tsunamis have a reflex reaction to tremors, which is to run to higher ground. Repeated false alarms due to earthquakes not generating significant tsunamis are perceived as a small cost compared with the potential benefit of saving lives one day. In modern industrialised societies, where the inconvenience of a false alarm is much harder to accept, the economic cost of false alarms should be quantitatively balanced against the safety benefits.

The quarter of a million who died from the Sumatra earthquake and tsunami of December 26, 2004 do not constitute the worst death toll in recent times. The 1976 Tangshan earthquake, which occurred shortly before the end of the Maoist era in China, was even more tragic. Ranking of the number of fatalities from earthquakes around the world yields a power-law known as Zipf's Law, whereby the frequency of events of rank k out of N is proportional to $1/k^S$. The normalisation factor is $\sum_{n=1}^{N} 1/n^S$. In the limit of infinite N, this is the zeta function $\varsigma(S)$. Zipf's Law applies to a multitude of disaster statistics, from the spread of forest fires to stock price plunges. Explorative analysis of Zipf's Law affords interesting mathematical opportunities for combining probability with number theory.

Fig. 2. On July 28, 1976, a massive earthquake destroyed the Chinese city of Tangshan, causing the highest earthquake death toll in modern times, exceeding a quarter of a million. [Photo from US Geological Survey photographic library]

Because of the fat tails of catastrophe loss distributions, it is hardly ever possible to identify and prepare for the worst event that might happen. Think of a historical Indonesian eruption worse than Krakatau in 1883, (which killed more than the Japanese tsunami of March 11, 2011), and you have Tambora in 1815, which caused mass starvation. Imagine an Indonesian eruption with a greater global impact than Tambora, and you have Toba, which decimated the human population 74,000 years ago. Funding for disaster preparedness, risk mitigation and resilience planning is strictly finite in every country; a serious misallocation of funding can divert resources away from urgent needs to those of lesser importance. On many issues, debates on the foibles of human behaviour may be rather academic, but on the mitigation of the risk from catastrophes, lives are at stake.

Throughout history, mathematicians have fought enlightenment battles against irrationality and illogical reasoning and thinking. Disaster prediction is one domain where irrationality has been rife, sometimes confounded with numerology. Featured in a 2009 Hollywood disaster movie, "Knowing", is the integer 911012996, which happened to be a fifty year-old time capsule future reference to the 2,996 victims of the terrorist attacks on September 11, 2001. Long integers do actually have a link with terrorism; not in forecasting disasters but in preventing them. The decryption of encoded messages sent by terrorist operatives is vital for interdicting plots early, before terrorists move towards their targets. With the prospect of communication interception, graph-theoretic analysis of the social networks of terrorists enables the interdiction likelihood to be calculated as a function of cell size. This is a neat piece of applied mathematics, but cryptography stands out as the quintessential area where man-made Asian catastrophes are being fought with mathematics.

Fig. 3. In 1883, a cataclysmic volcanic eruption occurred on the Indonesian island of Krakatau, which generated a giant tsunami which killed at least 36,000 people. [1888 lithograph]

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