Evolution of Tsunami Science

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1. Introduction

Although tsunamis have been leaving tragic traces in human history from ancient times, all earlier tsunami descriptions were based on anecdotal evidence of a few survivors, embedded in myths, folklore, and art. Thus, the classical Plato description of death of legendary Atlantis nowadays looks like a typical sequence of geological events triggered by a destructive earthquake and followed by a giant tsunami. More recent artwork by Hokusai (1760–1849) keeps inspiring tsunami scientists and wave professionals by the beauty and realism of depicted breaking waves.

In modern history, the Japan Meteorological Agency (JMA) initiated tsunami warning services in 1952, and NOAA's Pacific Tsunami Warning Centre (PTWC, Honolulu) was established in 1948 following the deadly 1946 Aleutian Island earthquake and tsunami. Until about 1980, semi-empirical charts (connecting tsunami threats to seismic sources) were the only quick forecasting tools available.

During the 80's and 90's, due to pioneering work of F. Imamura, N. Shuto, C. E. Synolakis and many others, fast computers and efficient models have been employed for tsunami modelling. In the early stages of the computing era, it was not possible to solve the two-dimensional Boussinesq equations with nonlinear and dispersion terms; instead, simplified alternatives became popular. In 1999, JMA has introduced the computer aided simulation system for quantitative tsunami forecasting, in which tsunami arrival times and heights are simulated and stored in the database for forecasting

Fig. 1. "Behind the Great Wave at Kanagawa" by Katsushika Hokusai (1760–1849).

tsunamis. The JMA has been further updating the system and now can issue the forecast 2 to 3 minutes after occurrence of an earthquake. Still, the 2011 Tōhoku earthquake and tsunami, which claimed 20,000 lives in Japan, wiped out several nearshore cities, and critically damaged Fukushima Nuclear Power Plant. The overall cost could exceed US$300 billion, making it the most expensive natural disaster on record.

Due to the half-century efforts by PTWC and JMA, most of the tsunami modelling and forecasting capabilities were focused on the Pacific Ocean; in other regions, tsunami science and awareness were not developed. Not surprisingly, the 2004 Indian Ocean Tsunami caught off guard the coastal communities along the Indian Ocean shores, killing almost 230,000 people.

The recent tragic events drew attention to the lack of tsunami-warning infrastructure, and triggered a worldwide movement to develop tsunami modelling and forecasting capabilities. The number of scientists and students migrating from different areas into the tsunami field has increased significantly, resulting in a re-examination of established approaches and perceptions, and in the development of novel ideas and methods. In Singapore, a similar movement has led to the development of national earthquake and tsunami predictive capabilities, and of a tsunami-warning system. This publication highlights some of the most important historical milestones that have led to our modern understanding of tsunami behaviour.

2. Soliton Theory

2.1. The first scientific encounter of solitons

One may start the description of tsunami behaviour using soliton theory, which is a simplified substitute for a full-scale tsunami model. In mathematics and physics, a soliton is a self-reinforcing solitary wave (a wave packet or pulse) that maintains its shape while it travels at constant speed. The soliton phenomenon was first described by John Scott Russell [12, 13] (Fig. 2), the British hydraulic engineer Scott Russell who observed a solitary wave in the Union Canal, Edinburgh (UK).

He reproduced the phenomenon in a wave tank (Fig. 3) and named it the "Wave of Translation" [13]. The discovery is described here in his own words:

Fig. 2. John Scott Russell (1808–1882).

"I was observing the motion of a boat which was rapidly drawn along a narrow channel by a pair of horses, when the boat suddenly stopped—not so the mass of water in the channel which it had put in motion; it accumulated round the prow of the vessel in a state of violent agitation, then suddenly leaving it behind, rolled forward with great velocity, assuming the form of a large solitary elevation, a rounded, smooth and well-defined heap of water, which continued its course along the channel apparently without change of form or diminution of speed. I followed it on horseback, and overtook it still rolling on at a rate of some eight or nine miles an hour [14 km/h], preserving its original figure some thirty feet [9 m] long and a foot to a foot and a half [300–450 mm] in height. Its height gradually diminished, and after a chase of one or two miles [2–3 km] I lost it in the windings of the channel.

Such, in the month of August 1834, was my first chance interview with that singular and beautiful phenomenon which I have called the Wave of Translation."

Following this discovery, Scott Russell built a 9 m wave tank in his back garden and made observations of the properties of solitary waves, with the following conclusions [12]:

- Solitary waves have the shape $a\,\mathrm{sech}^2(k(x-ct))$, where a is the wave height, k is the wave number, and c is the wave speed;
- A sufficiently large initial mass of water produces two or more independent solitary waves;
- Solitary waves can pass through each other without change of any kind;
- A wave of height a and travelling in a channel of depth h has a velocity given by the expression $c=\sqrt{g(a+h)}$, where g is the acceleration of gravity, implying that a large amplitude solitary wave travels faster than one of low amplitude.

Throughout his life Russell remained convinced that his "Wave of Translation" was of fundamental importance, but 19th and early 20th century scientists thought otherwise, partly because his observations could

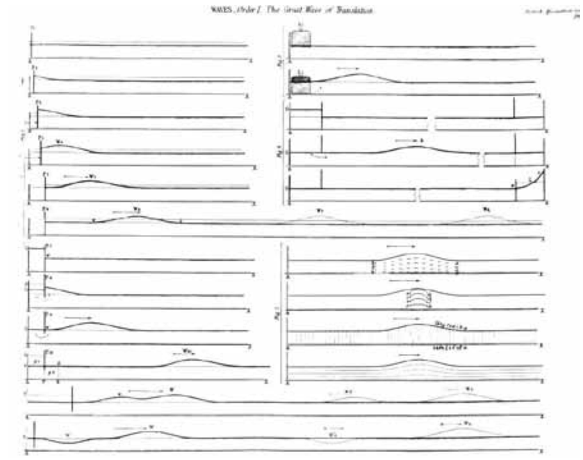


Fig. 3. Russell's (1844) wave tank to study solitons.

not be explained by the then-existing theories of water waves. Subsequently, the observations were reinforced by theoretical work of the French mathematician and physicist J. Boussinesq [3].

2.2. Behaviour of solitons

Soliton propagation could be understood by means of a simple convective wave equation

$$\eta_t + c\eta_x = 0 \tag{1}$$

where the wave speed $c=c(\eta,x,t)$ is a function of surface elevation η, space x, and time t.

If $c = const$, this equation has travelling wave solutions, and all waves propagate at the same speed c. Particular interest for the subsequent examples attaches to the initial condition illustrated in [4] at $t = 0$

$$\eta(x,0)=\mathrm{sech}^2(x)\,, \tag{2}$$

for which the exact solution of Eq. (1) at time t for $c = const$ is

$$\eta(x,t)=\mathrm{sech}^2(x-ct)\,. \tag{3}$$

Here $\mathrm{sech}(x)=1/\cosh(x)=2/(e^x+e^{-x})$.

If the wave speed is dependent on the wave elevation, $c=c(\eta)$, initial wave profiles are generally not self-preserving. The simplest example is given by $c=\eta$, which being substituted into the linear, non-dispersive wave Eq. (1) yields

$$\eta_t + \eta\eta_x = 0\,. \tag{4}$$

This equation governs a nonlinear wave propagation. Using the initial wave profile Eq. (2), solutions for $\eta(x,t)$ describe waves such that the profile eventually becomes multi-valued and gradient blowup occurs (Fig. 4a).

Dispersion behaviour of the waves is described with a dispersive wave equation

$$\eta_t + \eta_{xxx} = 0\,. \tag{5}$$

This equation has travelling wave solutions

$$\eta(x,t) = \int_{-\infty}^{\infty} a(k)\exp(ik^3t + ikx)dk$$

where $a(k)$ is the component amplitude of the Fourier transform of the initial profile. If the initial wave profile is again in the form of Eq. (2), one can observe that a single propagating wave splits (disperses) from the tail and resulting in oscillatory waves of different frequency that continue to propagate at different speed as in Fig. 4b. This behaviour is explicitly embedded in the dispersive wave solution depicting shorter harmonics (with larger k) propagating left relative the peak of the wave. Hence, the solutions $\eta(x,t)$ do not describe localised travelling waves of constant shape and speed.

Wave propagation exhibits both nonlinear and dispersive behaviour if described with the Korteweg–de Vries (KdV) equation:

$$\eta_t + \eta\eta_x + \eta_{xxx} = 0\,. \tag{6}$$

The equation is named after Korteweg and de Vries [9], though the equation was in fact first derived by Boussinesq [3]. This equation has localised travelling wave solutions (solitary waves) in the form of

$$\eta(x,t) = 3c\ \mathrm{sech}^2\left(\sqrt{(x-ct)}\,/\,2\right). \tag{7}$$

It was then understood that balancing dispersion against nonlinearity leads to travelling wave solutions (Fig. 4c) as earlier observed by Scott Russell, and this is precisely the physical feature of solitons.

For a tsunami propagating in the ocean, dispersion and nonlinearity are not necessarily in equilibrium. In somewhat simplistic terms, if nonlinearity dominates (usually nearshore) the incident soliton tends to break from the front side; whereas in deepwater conditions dispersion results in the soliton shedding waves from the tail. A tsunami can propagate across the ocean as a series of several solitons probably originating from a single wave at source.

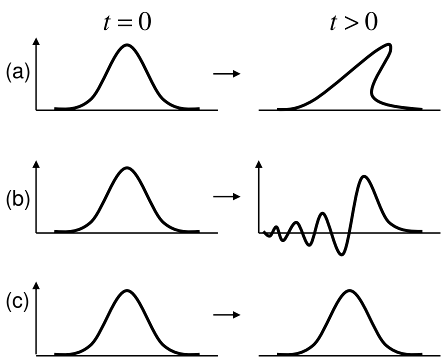


Fig. 4. Nonlinear and dispersive soliton behaviour: (a) nonlinear term only; (b) dispersion term only; (c) nonlinear and dispersion terms balanced together.

3. Boussinesq-type Equations

To draw a more complete and accurate picture of tsunami behaviour in the ocean, one can use the nonlinear water-wave model involving Laplace's equation combined with boundary conditions, nonlinear at the free-surface and linear at the sea bottom [5], which can be rewritten in dimensionless form as

$$\delta(\varphi_{xx} + \varphi_{yy}) + \varphi_{zz} = 0 \ \text{ in fluid} \tag{8}$$

$$\varphi_t + \frac{\varepsilon}{2}(\varphi_x^2 + \varphi_y^2) + \frac{\varepsilon}{2\delta}\varphi_z^2 + \eta = 0 \ \text{ at } z = \varepsilon\eta \tag{9}$$

$$\delta[\eta_t + \varepsilon(\varphi_x\eta_x + \varphi_y\eta_y)] - \varphi_z = 0 \ \text{ at } z = \varepsilon\eta \tag{10}$$

$$\varphi_z = 0 \ \text{ at } z = -1. \tag{11}$$

Here φ is the velocity potential, giving fluid velocity components $u = \frac{\partial\varphi}{\partial x}$, $v = \frac{\partial\varphi}{\partial y}$, $w = \frac{\partial\varphi}{\partial z}$ and $\eta(x,y,z,t)$ is the free surface. The scale parameters $\varepsilon = a/h$ and $\delta = h^2/l^2$ are introduced to represent nonlinearity and dispersion, respectively. The horizontal length-scale of the sea bed non-uniformities L is assumed to be much larger than the wave length l (i.e., $\gamma \equiv l/L$, $\gamma << 1$), resulting in the sea bed being "*mild slope*", and the gradient of the sea-bed shape being neglected.

For the 2004 Indian Ocean Tsunami, a = 1 m in the ocean, and up to 10 m nearshore; h = 4000 m and 10 m, respectively; l = 400 km and 50 km, respectively. Thus, the introduced scale parameters may have ranges: $\varepsilon = 10^{-4}$ in the ocean and up to 1 nearshore; $\delta = 10^{-4}$ and 10^{-5}, respectively.

Integration of Eqs. (8)–(11) is complicated by the fact that the moving surface boundary is part of the solution. Direct numerical methods for solving the full equations exist, but they are extremely time-consuming and can only be applied to small-scale problems. As it is currently impracticable to compute a full solution valid over any significant domain such as the entire Indian or Pacific Ocean, approximations must be adopted, including the so-called Boussinesq-type formulations of the water-wave problem.

Following Boussinesq [2], we expand the velocity potential in terms of δ without any assumption about ε:

$$\varphi = \varphi_0 + \delta\varphi_1 + \delta^2\varphi_2 + \cdots \tag{12}$$

and substitute into Eqs. (8)–(11) to derive the unknown terms $\varphi_0, \varphi_1, \varphi_2$.

The idea behind the Boussinesq approximation (12) was to incorporate the effects of non-hydrostatic pressure, while eliminating the vertical coordinate z, thus reducing the computational effort relative to the full three-dimensional problem. The assumption that the magnitude of the vertical velocity increases polynomially from the bottom to the free surface (Fig. 5) inevitably leads to some form of depth limitation in the accuracy of the embedded dispersive and nonlinear properties. Hence, Boussinesq-type equations are conventionally associated with relatively shallow water.

Let us retain all terms up to order δ, ε in Eq. (9) and $\delta^2, \varepsilon^2, \delta\varepsilon$ in Eq. (10) to obtain 2-D Boussinesq-type equations

$$\eta_t + (u(1+\varepsilon\eta))_x + (v(1+\varepsilon\eta))_y - \frac{\delta}{6}((\nabla^2 u)_x + (\nabla^2 v)_y) = 0 \quad (13)$$

$$u_t + \varepsilon(uu_x + vu_y) + \eta_x - \frac{1}{2}\delta(u_{txx} + v_{txy}) = 0 \quad (14)$$

$$v_t + \varepsilon(uv_x + vv_y) + \eta_y - \frac{1}{2}\delta(u_{txy} + v_{tyy}) = 0\,. \quad (15)$$

To simplify the set of Eqs. (13)–(15) to a single one, we assume a similar small scale for the introduced parameters, i.e., $\delta \sim \varepsilon$; retain only one dimension (x-dependence); eliminate u in linear terms of Eq. (13) using Eq. (14), and in nonlinear terms using linearised relationship $u = \eta + O(\gamma)$.

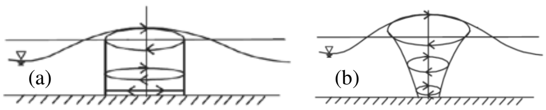


Fig. 5. Vertical structure of the water column beneath the waves: (a) hydrostatic assumption; (b) non-hydrostatic assumption.

Resulting expression (the Boussinesq equation) comprised of second and higher order derivatives, can be simplified further by letting $\delta \sim \varepsilon \sim \gamma$. In physical terms the assumption $\gamma << 1$ impose wave parameters, such as height, length and direction of propagation, to be slow varying at a distance of the wave length. In contrast with the Boussinesq equation, the condition allow to consider the progressive wave solution travelling to one direction only, positive or negative with respect to x direction. For the positive direction we obtain a single equation, universally known as the Korteweg and de Vries (KdV)

$$\eta_t + \left(1 + \frac{3}{2}\varepsilon\eta\right)\eta_x + \frac{1}{6}\delta\eta_{xxx} = 0\,. \quad (16)$$

While deriving Eqs. (13)–(15) we have implicitly assumed that $\delta << 1$, $\varepsilon << 1$, $\gamma << 1$ and $\delta \sim \varepsilon$; therefore, the Boussinesq equations include only the lowest-order effects of frequency dispersion and nonlinearity. They can account for transfer of energy between different frequency components, changes in the shape of the individual waves, and the evolution of wave groups in the shoaling irregular wave train. However, the standard Boussinesq equations have two major limitations in their application to long waves on shallow water:

- the depth-averaged model describes poorly the frequency dispersion of wave propagation at intermediate depths and deep water;
- the weakly nonlinear assumption is valid only for waves of small surface slope, and so there is a limit on the largest wave height that can be accurately modeled.

Modern tsunami research experiences two contradictory trends, one is to include more physical phenomena (previously neglected) into consideration, and another is to speed up the code to be used for the operational tsunami forecast.

Although higher-order Boussinesq equations for the improvement of the description of nonlinear and dispersive properties in water waves have been attempted and have been successful in certain respects, most of these attempts have involved numerous additional derivatives and hence made the accurate numerical solution increasingly difficult to obtain. In justification of such derivations of higher-order terms in the Boussinesq equations, preference has often been given to artificially constructed test cases having little (if any) correspondence with real tsunamis. The Northern Sumatra (December 2004) tsunami had provided a new test case for the various models. After several decades of intensive worldwide research, it is interesting to read the conclusion of Grilli *et al.* [7] that "*… in view of the apparently small dispersive effects, it could be argued that the use of a fully nonlinear Boussinesq equation model is overkill in the context of a general basin-scale tsunami model. However, it is our feeling that the generality of the modelling framework provided by the model is advantageous in that it automatically covers most of the range of effects of interest, from propagation out of the generation region, through propagation at ocean basin scale, to runup and inundation at affected shorelines.*"

Even the presence of the third-order derivative terms for dispersion in the standards Boussinesq Eqs. (13)–(15) is considered challenging enough to be omitted in popular operational tsunami modelling codes, such as Tunami-N2 [6, 8].

Boussinesq equations with omitted dispersion terms often are referred to as the Nonlinear Shallow Water Equations (NSWE). Alternative simplification suggested in MOST [15] and COMCOT [10] is to use NSWE, but implicitly include dispersion phenomenon by shaping a numerical approximation error in the form of the third-order derivatives (dispersion terms).

The optimal code for tsunami modelling must be sufficiently fast and accurate; however, the notions of speed and accuracy are quickly changing to reflect

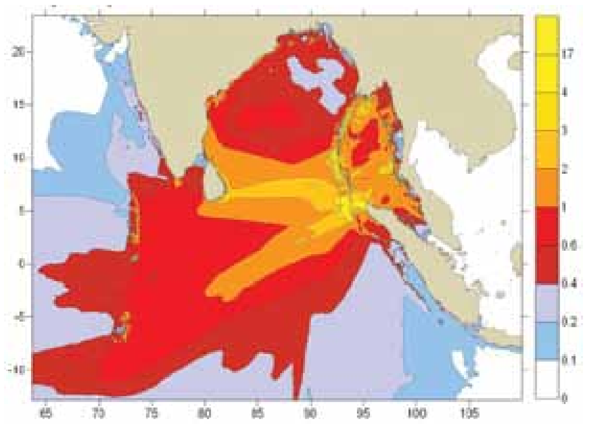


Fig. 6. Maximum wave height computed for the 2004 Indian Ocean Tsunami [4].

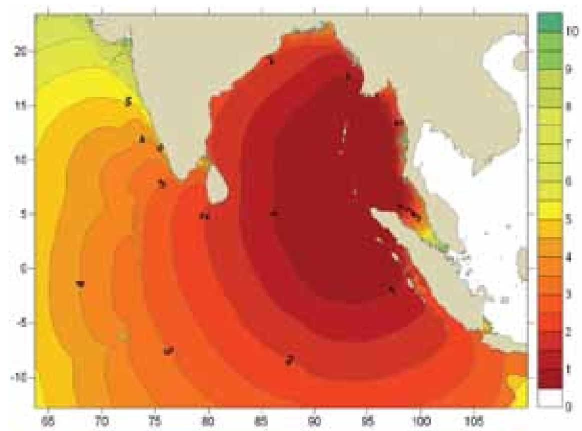


Fig. 7. Arrival time of first wave computed for the 2004 Indian Ocean Tsunami [4].

current understanding of tsunami physics as well as growing computational power. Hence, in order to assess parameters of the currently optimal code, established and new approaches need to be regularly re-evaluated to ensure that the most important (and yet computationally affordable) phenomena are taken into account. The importance of some phenomena, potentially capable of affecting tsunami propagation characteristics, has been quantitatively evaluated by Dao and Tkalich [4]. Computations (Figs. 6 and 7) show that the following phenomena have been important for the Northern Sumatra Tsunami (in reducing order of importance):

- Astronomical tide at high phase may lead to nonlinear increase of the tsunami height up to 0.5 m during high tide and increase arrival time by 30 minutes;
- Reduction of bottom friction lead to increases of 0.5–1.0 m in the maximum tsunami height nearshore, where as in the deep ocean the effect of bottom friction is negligible;
- Dispersion effects have significant influence in the deep ocean, leading to a drop of 0.4 m (40%) in the computed maximum tsunami height. No significant change in arrival time is observed.

To avoid complex derivatives and unnecessary complications posed by the Boussinesq model, Stelling and Zijlema [14] proposed a semi-implicit finite difference model, which accounts for dispersion through a non-hydrostatic pressure term. In both, the depth-integrated and multi-layer formulations, they decompose the pressure into hydrostatic and non-hydrostatic components. The solution to the hydrostatic problem remains explicit; the non-hydrostatic solution derives from an implicit scheme to the 3-D continuity equation. The depth-integrated governing equations are relatively simple and analogous to the nonlinear shallow-water equations with the addition of a vertical momentum equation and non-hydrostatic terms in the horizontal momentum equations. Numerical results show that both depth-integrated models estimate the dispersive waves slightly better than the classical Boussinesq equations.

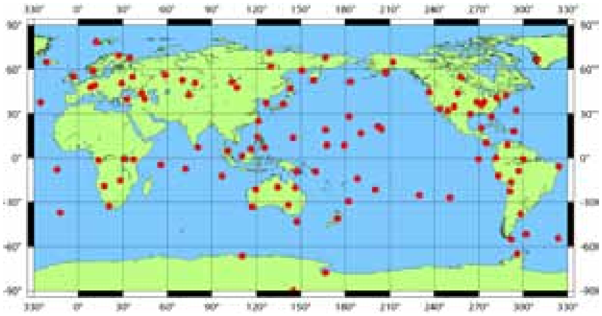
4. Tsunami Warning

Long before the modern instrumental era, people were trying to predict earthquakes and tsunamis using various nonscientific means (i.e., all that was then available). In Japan, one of the earliest forms of tsunami warning is literally cast in stone. "If an earthquake comes, beware of tsunamis," and "Remember the calamity of the great tsunamis. Do not build any homes below this point," read stone slabs, hundreds such markers doted Japanese coastline, some more 600 years old.

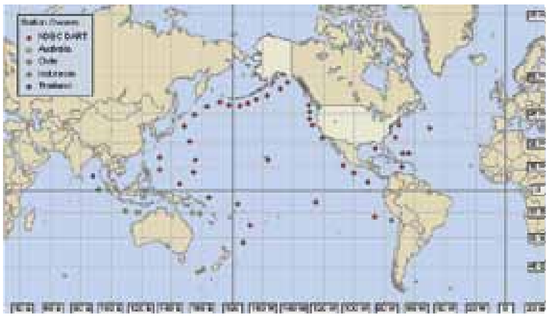
Nowadays, many scientifically-based methods of Earth observation are sufficiently developed and utilised [1], or could be developed in a short time-frame [11].

As most tsunamis are triggered by earthquakes, seismometers are the first obvious choice to trigger a tsunami warning system and to estimate the source parameters. Seismic signals from the near-real-time IRIS Global Seismographic Network (Fig. 8a) are commonly used to infer an earthquake's magnitude and epicenter location. If a tsunami has been generated, the waves propagate across the ocean eventually reaching one of the NOAA-developed DART buoys (Fig. 8b), which report sea-level measurements back to the tsunami-warning centre.

Two auxiliary sources of tsunami information have to be mentioned, i.e., near-shore tide gauges and open-sea satellite altimetry. The tide gauge measurements are complicated by variations in local bathymetry and harbour shapes, which severely limit the effectiveness of the data for providing useful measurements for tsunami forecasting. Tide gauges can provide verification of tsunami forecasts, but they cannot provide the data necessary for efficient forecast itself, and definitely not for the coast where they are installed. Tsunami detection by satellite altimetry is currently restricted by the high cost of imaging and low frequency of sampling.



(a) IRIS Global Seismographic Network



(b) Deep-ocean Assessment and Reporting of Tsunamis (DART) global map

Fig. 8. Existing observation networks.

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Pavel Tkalich has been trained in applied mathematics and cybernetics in Kiev University (Ukraine). His PhD dealt with linear and nonlinear water waves, including soliton-type solutions of the Bousinesq equations. Starting from water quality impact assessment after the Chernobyl Accident, Tkalich's interests have extended to modelling of algal blooms, oil spills, and other pollutants in the aquatic environment. The latest areas of research include also sea level rise and extremes, projections of storm surges and wind waves with respect to the climate change. Since repatriation to Singapore, Pavel Tkalich has been leading a number of national-importance projects, including development of Singapore Tsunami Warning System and Climate Change Vulnerability Study.